

This electronic thesis or dissertation has been downloaded from the King's Research Portal at <https://kclpure.kcl.ac.uk/portal/>



Making sense of metaphysical modal claims

Mankowitz, Poppy Rebecca Mary

Awarding institution:
King's College London

The copyright of this thesis rests with the author and no quotation from it or information derived from it may be published without proper acknowledgement.

END USER LICENCE AGREEMENT



Unless another licence is stated on the immediately following page this work is licensed

under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International

licence. <https://creativecommons.org/licenses/by-nc-nd/4.0/>

You are free to copy, distribute and transmit the work

Under the following conditions:

- Attribution: You must attribute the work in the manner specified by the author (but not in any way that suggests that they endorse you or your use of the work).
- Non Commercial: You may not use this work for commercial purposes.
- No Derivative Works - You may not alter, transform, or build upon this work.

Any of these conditions can be waived if you receive permission from the author. Your fair dealings and other rights are in no way affected by the above.

Take down policy

If you believe that this document breaches copyright please contact librarypure@kcl.ac.uk providing details, and we will remove access to the work immediately and investigate your claim.

Making Sense of Metaphysical Modal Claims
MPhilStud, Department of Philosophy, King's College London

Poppy Mankowitz

1st September 2014

Abstract

In my thesis, I will consider the topic of metaphysical modal claims. The problem with metaphysical modal claims is that it is not entirely clear how one goes about interpreting and assigning truth-values to some of them. It shall be argued that a flexible, reductive account of the way in which speakers issue and interpret modal claims would address this sense of confusion.

I will therefore motivate, develop and defend a reductive, linguistic ersatz account in a deliberately simplified language of modal predicate logic. This account will construe possible worlds as maximal-consistent sets of sentences, defining these structures in such a way that modal notions are not employed. The idea of assumption-relative modal operators and accessibility relations will then be defined, in order to accommodate the fact that most modal claims do not concern broad logical possibility. Finally, a means of incorporating essentialist claims into assumption-sets shall be developed, in order to handle metaphysical modal claims in particular. An essentialist predicate must be construed non-modally in order to avoid compromising the reductive account.

The developed account will also be assessed. I will argue that it is indeed reductive, that Lewis's criticism of reductive linguistic ersatz accounts do not apply in this case, and that no other means of constructing possible worlds has the potential to give a similar reductive account. However, the commitment to a primitive essentialist predicate undermines the motivation for the proposal *qua reductive* account. I will consider this objection, in addition to others, and reiterate the theoretical benefits of the account.

Acknowledgments

First and foremost, I thank my supervisor Wilfried Meyer-Viol. He has been extremely generous with his time, and has patiently helped me with both the technical features and the philosophical ideas within my thesis. I have greatly enjoyed our brainstorming sessions.

I am also extremely grateful to Shalom Lappin for many useful discussions about the semantics of epistemic modals and the importance of partial worlds. My conversations with Chris Hughes about the metaphysics of possible worlds and the limitations of modal reductivism helped me formulate the initial ideas for the thesis, and I thank him too. Finally, I wish to express my thanks to the staff and students who commented on the work I presented at the Advanced Research Seminar at King's, the Language and Cognition Research Seminar at King's, the NIP Relativism Masterclass at Aberdeen in July 2013, and the Logic, Grammar and Meaning conference at the University of East Anglia in June 2014.

Contents

Introduction	3
0.1 Modal Predicate Logics	5
0.1.1 Introduction to Modal Predicate Logics	5
0.1.2 Modal Predicate Logics and Metaphysical Modal Claims	9
0.2 Motivating a Reductive Account of Modal Terms	11
0.2.1 Reductive and Primitive Definitions	12
0.2.2 Two Principles Motivating Modal Reductivism	13
0.2.3 Precising the Principle of Limiting Primitives	15
0.3 The Type of Reductive Account to Pursue	21
0.3.1 Different Ways of Defining ‘Possible World’	21
0.3.2 Linguistic Ersatzism as a Reductive Account	25
0.4 A Reductive Linguistic Ersatz Account	28
0.4.1 A Non-circular Definition of ‘Possible World’	28
0.4.2 Lewis’s Material-Adequacy Objection	30
0.4.3 Relative Accessibility Relations	33
0.4.4 A Materially-Adequate Definition of ‘Possible World’	37
0.5 Extending the Account to Metaphysical Modal Claims	41
0.5.1 Attempt 1: The Current Account Unmodified	41
0.5.2 Attempt 2: Introducing Constants into the Account	43
0.5.3 Attempt 3: Introducing an Essentialist Predicate into the Account	46
0.6 Accepting \mathbb{E} as a Primitive	49
0.6.1 Essential Properties: Traditional and Finean Accounts	49
0.6.2 Defending the Irreducibility of \mathbb{E} to Modal Terms.	52
0.6.3 Defending the Material-Adequacy of the Traditional Definition	55
0.6.4 The Unwelcome Consequences of Taking \mathbb{E} as Primitive	58
0.7 Objections and Replies	60
Conclusion	82

Introduction

The aim of this dissertation is to determine the meanings and truth-values of metaphysical modal claims. By a *modal claim* I mean either a sentence type or sentence token (that is, a type issued in a context) that contains a modal term. A *modal term* is ‘any word or phrase that can be applied to a given statement S to create a new statement that makes an assertion about the mode of truth of S: about when, where or how S is true, or about the circumstances under which S may be true.’¹ Another way of characterising modal terms is through noting the fact that their presence causes sentences to make reference to ‘situations which need not be real’.² Within the literature, different ‘*flavours*’ of modality are often identified. The most common flavours include *logical*, *analytic*, *deontic*, *epistemic*, *nomological* and *metaphysical* modal claims, with the latter variety being the topic of the current dissertation. It is difficult to give a precise definition of ‘metaphysical’ modal claims. Examples of modal claims that are generally considered metaphysical may be given, however. This will serve to illustrate why there might be questions about their meanings and truth-values.

Take the actual individual Adam, complete with a set of actual properties. Which of these properties could Adam have lacked, and which other properties could he have possessed, whilst still remaining Adam? Could he have had a different name?³ Could he have died a year later than he in fact did?⁴ Could he have been created from different gametes?⁵ Could he have possessed all of the properties that are possessed by another actual individual, Noah?⁶ Could he have been an angel, or a poached egg?⁷ These all seem to be metaphysical modal questions, and to affirm or deny any one of them would be to make a metaphysical modal claim. In addition, more general claims such as ‘All humans necessarily derive from their actual gametes’ would seem to count as metaphysical modal claims. In short, a modal claim seems ‘metaphysical’ by virtue of having an intuitively metaphysical subject matter, though it is difficult to be precise about what this consists in.

The trouble with metaphysical modal claims such as these is that their meanings and truth-values are obscure: for it is not clear what could make

¹Goldblatt (2006) pp.2-3.

²Portner (2009) p.1.

³Chisholm (1967) p.6.

⁴*Ibid.* p.1.

⁵Kripke (1980) p.112.

⁶Chisholm (1967) pp.3-4.

⁷Lewis (1986) p.251.

such claims true or false. The task, then, shall be to make sense of metaphysical modal claims.

Some general aims relating to this task will be as follows: firstly, the meanings and truth-values of metaphysical modal claims should be *explained within an account of non-metaphysical modal claims*. One reason for this requirement is that the meanings and truth-values of some *non*-metaphysical modal claims seem as obscure as those of the metaphysical variety. Furthermore, categorising metaphysical modal claims and non-metaphysical modal claims as ‘*modal*’ indicates that they share some common features, which in turn suggests that a similar account of their meanings would be appropriate. Secondly, the account should *reflect the process by which speakers actually issue and interpret modal claims*. Thirdly, I shall aim to develop an account of modal claims that is *flexible*, allowing it to be adapted to accommodate diverse and sometimes conflicting philosophical perspectives. This will involve my refraining from taking a position on matters that the account can afford to remain neutral on. The flexibility of the account is intended to maximize its appeal and utility.

As a first attempt at achieving this task, the language of *modal predicate logics* shall be defined in §1.1. In §1.2, it will be found that translating metaphysical modal claims into such a language fails to sufficiently clarify their meanings and truth-values; though dealing with the deliberately simplified language of modal predicate logic will turn out to be a useful means to analyse metaphysical modal claims whilst avoiding paying attention to the complexities of the syntax of natural languages. Having established that more should be said about the nature of possible worlds than modal predicate logic reveals, I will motivate a reductive account of metaphysical modal claims. In §2.1, reductivism shall be contrasted with primitivism. Then, two principles shall be discussed that seem to provide the best motivation for modal reductivism, in §2.2-3. The next step will involve establishing exactly what type of reductive account we should pursue. In §3.1, the different definitions of ‘possible world’ that are typically available will be listed. In §3.2, it will be argued that only linguistic ersatzer accounts possess the potential to be reductive. This brings us to the point at which a reductive, linguistic ersatzer account needs to be provided. In §4.1, ‘possible world’ shall be defined through making use of the non-modal terms ‘maximal’ and ‘consistent’. An objection from Lewis towards this sort of approach will be considered in §4.2. This will motivate the introduction of assumption-relative accessibility relations and modal operators in §4.3. In §4.4, it will be argued that this linguistic ersatzer account is indeed reductive. A flexible, reductive account has thus been provided; however, it is not obvious how to use it to interpret

metaphysical modal claims. In order to extend the account to metaphysical modal claims, I shall consider the implications of refraining from modifying the current account in §5.1. Having noted that this approach fails, I shall consider implementing the minimal modification of introducing constants into the modal predicate language in §5.2. Yet in §5.3, it will be found that the only way to accommodate metaphysical modal claims is to implement the more significant modification of postulating that an essentialist predicate appears in the assumption-sets for such claims. The traditional and Finean views of essentialist predicates shall be discussed in §6.1. In §6.2 it will be argued that, as Fine claims, modal terms cannot be employed to give a reductive definition of this essentialist predicate; yet in §6.3 I will argue, contra Fine, that an analysis of the essential predicate in terms of modal expressions will at least be materially-adequate. Then, in §6.4, I consider the unwelcome consequences of taking the essentialist predicate as a primitive, and assess the implications for my account. §7 is reserved for addressing several objections.

0.1 Modal Predicate Logics

In §1.1, I shall begin the task of making sense of metaphysical modal claims by defining a class of modal predicate logics. In §1.2, the meanings of metaphysical modal claims translated into a language of a modal predicate logic shall be found to be clarified only to a limited degree by this translation process. It will be concluded that the truth-conditions of modal expressions must be further clarified, and the term ‘possible world’ defined, in order to shed light on metaphysical modal claims.

0.1.1 Introduction to Modal Predicate Logics

A *modal logic* allows a formula to be assigned truth-values at multiple points of evaluation, in addition to allowing the truth-values that a formula takes at alternate points of evaluation to be specified at a single point of evaluation. This contrasts with classical logics, in which fixed truth-values are assigned to formulas relative to an interpretation. Modal logics therefore provide a means to qualify the truth of a sentence, since a host of counter-factual truth-values may be expressed for that sentence. The connection between modal logics and modal claims should be clear, given that the latter were said to pertain to situations which need not be real.

Modal logics will prove to be useful machinery in the investigation of metaphysical modal claims, so must be characterised before they can rea-

sonably be applied.⁸ The language of modal predicate logics consist of:

1. a finite set of individual variables
2. the standard predicate logic connectives \neg , \wedge and the quantifier \exists ; along with the identity sign $=$
3. a finite set of n -place predicate symbols
4. the unary modal operator \Diamond

One of the modal operators is here being taken as primitive within the language of modal predicate logic, with the other being definable in terms of it; that is, $\Box P = \neg\Diamond\neg P$, and $\Diamond P = \neg\Box\neg P$, in accordance with Aristotle's Modal Square of Opposition. It is the modal operators that distinguish modal predicate logic from standard predicate logic, with \Box normally being interpreted as 'it is necessary that...' and \Diamond normally interpreted as 'it is possible that...'.⁹

The *atomic formulas* of a modal predicate logic language are of the form $P(x_1...x_n)$ or $y = z$, where $x_1...x_n, y$ and z are individual variables and P is an n -place predicate. The definition of the set of *well-formed formulas* of modal predicate logic proceeds thus:

1. all atomic formulas are *wffs*
2. if ϕ, ψ are *wffs*, and α is an individual variable, then: $\neg\phi$, $\phi \wedge \psi$, $\exists\alpha\phi$ and $\Diamond\phi$ are also *wffs*
3. nothing else is a *wff*

It will be useful to isolate the non-modal component of such a language. For a modal predicate logic language MPL' consisting of certain variables, predicates, connectives, quantifiers and modal operators, let a language $MPL'_{-\Box, -\Diamond}$ be MPL' without its modal operators. Then define the well-formed formulas of $MPL'_{-\Box, -\Diamond}$ in an identical way to the definition of *wffs* for the language MPL' , except for omitting the clause concerning modal operators. The set of *wffs* of $MPL'_{-\Box, -\Diamond}$ will therefore be a proper subset of the set of *wffs* of MPL' .

⁸The following is adapted from: Fitting and Mendelsohn (1998).

Modal predicate logics contain every standard axiom of predicate logics.⁹ In addition, every modal predicate logic includes the *K* axiom $\Box(P \rightarrow Q) \rightarrow (\Box P \rightarrow \Box Q)$. A range of other axioms governing the behaviour of the modal operators may also be included, and different systems of modal logic result from the addition of different axioms. This completes the syntactic characterisation of modal predicate logics.

In order to interpret modal logics, machinery developed in Kripke (1963) is used. A *Kripke frame* for a modal predicate logic is a triple $\langle W, R, D \rangle$, where W is a set of worlds, R is a binary relation on W and D is a set of individuals. Each feature of a frame shall be discussed in turn.

W consists of what I earlier referred to as ‘points of evaluation’. However, these points of evaluation are often referred to as ‘possible worlds’ or just ‘worlds’. This derives from Leibniz’s work on necessity and contingency, which made use of the notion of the unactualized, merely possible worlds that God could have created.¹⁰ The reason for construing of the points of evaluation as worlds can be seen once it is realised that the classical logical laws of non-contradiction and excluded middle hold within all normal modal predicate logics. The law of non-contradiction is of the form $\neg(P \wedge \neg P)$, and asserts that a *wff* cannot simultaneously hold and not hold. The law of excluded middle is of the form $(P \vee \neg P)$, and requires that a *wff* must either hold or not hold. Conditions are therefore imposed such that every *wff* of a modal predicate logic language must be either true or false, but not both true and false, at each point of evaluation. Interpreting the points of evaluation relative to which formulas are assigned truth-values as *worlds* is therefore natural, due to the fact that each point represents a non-contradictory and complete state of affairs (with respect to a language). It follows that, with a rich enough language, both the actual world and a range of (logically consistent) counter-factual worlds may be described within modal logic.

Without an accessibility relation R , the modal operators range over the entire set of worlds W . This means that a claim is necessary if and only if it holds at every world of W . The axioms 5 and T ($\Diamond P \rightarrow \Box \Diamond P$ and $\Box P \rightarrow P$, respectively) would therefore be valid on every frame, meaning that only the system of modal logic S5 can be captured. Yet it was earlier mentioned that different axioms may be adopted in order to generate different systems of modal logic. The semantics of modal logic must be able to reflect these diverse axioms if they are to provide an interpretation of the full range of

⁹These include the standard axiom schemata of propositional logic (see Kleene (2002) pp.15-16) in addition to two axiom schemata governing the behaviour of the quantifiers (*Ibid.* pp.94-5).

¹⁰For example, see: Leibniz (1985).

modal logics. With the introduction of an accessibility relation, a ‘relative’ notion of possible worlds results, in which a particular world may or may not be possible relative to another world.¹¹ This is achieved by defining different conditions on an accessibility relation (e.g. reflexivity, transitivity etc.), which correspond to certain axioms (respectively, $\Box P \rightarrow P$, $\Box\Box P \rightarrow \Box P$) and result in semantics for distinct modal logics (respectively, T, K4). If a world Δ is accessible from Γ , then we write $\Gamma R \Delta$ or, alternatively, $\langle \Gamma, \Delta \rangle \in R$.

The domain function D associates with each world the set of individuals that exist at that world. The domain of the frame $D(F) = \cup\{D(\Gamma) | \Gamma \in W\}$. $\langle W, R, D \rangle$ is a *constant domain frame* if $D(\Gamma) = D(\Delta)$ for all $\Gamma, \Delta \in W$. Otherwise, $\langle W, R, D \rangle$ is a *varying domain frame*. For a constant domain frame, $D(F) = D(\Delta)$ for any $\Delta \in W$; whereas for a varying domain frame, this cannot be guaranteed. In words, two sorts of frames may be defined: ones for which there is a single domain for any model based on that frame, and ones for which the set of individuals assigned to each world of a model is permitted to vary. This completes the informal explanation of Kripke frames.

As things currently stand, there is no way to interpret and assign truth-values to formulas. A frame must therefore be extended into a *Kripke model* $\langle F, l \rangle$, where F is a Kripke frame and l is an interpretation function. The interpretation function assigns to each n -place predicate P , and to each world $\Gamma \in W$, some relation on the domain D of the frame. In other words, l gives the extension of every predicate relative to each world of a frame. In addition, a *valuation function* is required, which is a function v that assigns to each free variable x some member $v(x)$ of $D(F)$ (for constant and varying domains alike). If v and w are two valuations, then w is an *x -variant* of v if and only if w and v agree on the assignments of all variables except possibly x ; and w is an *x -variant* of v at Γ if and only if w is an *x -variant* of v and $w(x) \in \Gamma$. The truth-conditions for modal predicate logics may now be defined.

Given a model M , for each $\Gamma \in W$ and each valuation v :

1. $M, \Gamma \Vdash_v P(x_1 \dots x_n) \iff \langle v(x_1) \dots v(x_n) \rangle \in l(P, \Gamma)$
2. $M, \Gamma \Vdash_v (x = y) \iff v(x) = v(y)$
3. $M, \Gamma \Vdash_v \neg \phi \iff M, \Gamma \not\Vdash_v \phi$
4. $M, \Gamma \Vdash_v \phi \wedge \psi \iff M, \Gamma \Vdash_v \phi \text{ and } M, \Gamma \Vdash_v \psi$

¹¹Kripke (1963) p.70.

5. $M, \Gamma \Vdash_v (\exists x)\phi \iff \text{for some } x\text{-variant } w \text{ of } v \text{ at } \Gamma, M, \Gamma \Vdash_w \phi$
6. $M, \Gamma \Vdash_v \Diamond\phi \iff \text{there is some } \Delta \in W : \Gamma R \Delta \text{ and } M, \Delta \Vdash_v \phi$

The other connectives, quantifier and operator may now be seen as abbreviations as follows:

- $M, \Gamma \Vdash_v \phi \vee \psi \iff M, \Gamma \Vdash_v \neg(\neg\phi \wedge \neg\psi)$
- $M, \Gamma \Vdash_v \phi \rightarrow \psi \iff M, \Gamma \Vdash_v \neg(\phi \wedge \neg\psi)$
- $M, \Gamma \Vdash_v \phi \leftrightarrow \psi \iff M, \Gamma \Vdash_v \neg(\phi \wedge \neg\psi) \wedge \neg(\neg\phi \wedge \psi)$
- $M, \Gamma \Vdash_v (\forall x)\phi \iff M, \Gamma \Vdash_v \neg(\exists x)\neg\phi$
- $M, \Gamma \Vdash_v \Box\phi \iff M, \Gamma \Vdash_v \neg\Diamond\neg\phi$

This completes the introduction to the syntax and semantics of first-order modal predicate logics. It is important that such logics have been rigorously defined, since the formal truth-conditions for the modal operators will later act as a point of departure in determining the meaning of metaphysical modal expressions in both logical and natural languages.

0.1.2 Modal Predicate Logics and Metaphysical Modal Claims

The current section will be concerned with applying the characterisation of modal predicate logics from the previous section to the issue of metaphysical modal claims. The aim will be to establish whether the formal truth-conditions allow us to understand the meaning and determine the truth-values of metaphysical modal claims as expressed in a modal predicate logic language. It will be shown that, although certain metaphysical claims may come out as true or false when translated into such a language, this is only the case for the logical *theorems* of that system. Nevertheless, attempting to understand metaphysical modal claims by dealing with their modal predicate logic translations will turn out to be a useful approach.

Let us start with the metaphysical modal claim ‘There is at least one individual who is necessarily human’, and consider the modal predicate logic translation $(\exists x)\Box(Human(x))$. The question we are seeking to answer is whether the definition of modal predicate logics helps us to interpret and assign a truth-value to such a claim. There are three ways in which the definition of modal predicate logic helps us. Firstly, *the logical truth-conditions of a particular metaphysical claim can be given*. For example:

1. $M, \Gamma \Vdash_v (\exists x)\Box(Human(x)) \iff M, \Gamma \Vdash_{v'} \Box(Human(x))$ for some x -variant v' of v at Γ

\iff for all $\Delta \in W$, if $\Gamma R \Delta$ then $M, \Delta \Vdash_{v'} (Human(x))$

$\iff v'(x) \in l(Human, \Delta)$

Secondly, we may *determine the logical implications of the truth of such a metaphysical modal claim within all modal logics*. For example, the inter-definability of the modal operators yields the following:

2. $M, \Gamma \Vdash_v (\exists x)\Box(Human(x)) \iff M, \Gamma \Vdash_v (\exists x)\neg\Diamond\neg(Human(x))$

Thirdly, we may *determine the logical implications of the truth of the claim within a particular system of modal logic*. For example, if we assess the metaphysical modal claim in a model based on a reflexive frame, then the following is T-provable:

3. $M, \Gamma \Vdash_{v'} \Box(Human(x)) \iff M, \Gamma \Vdash_{v'} (Human(x))$

Results such as the three detailed above may demonstrate some collection of metaphysical modal claims to be inconsistent in all or some modal logics, in addition to giving the truth-conditions of metaphysical modal claims expressed in a modal predicate logic language. However, modal predicate logics have nothing to say about whether claims such as ‘There is at least one individual who is necessarily human’ are actually *true*.

This is the case because the value of the claim ‘There is at least one individual who is necessarily human’ depends crucially on the specification of a model and a world of evaluation. No guidelines regarding the types of models we should specify for translations of metaphysical modal claims has been provided. Yet this has important implications for the process of determining whether or not such claims are actually true. For example, a suitable model to assign a truth-value to the metaphysical modal claim could be set up as follows: let $M = \langle W, R, D, l \rangle$, where $W = \{w_1, w_2, w_3\}$, $R = (\langle w_1, w_2 \rangle, \langle w_2, w_3 \rangle)$ and $D(w_1, w_2, w_3) = (a)$. Suppose $l(Human, w_2) = l(Human, w_3) = \{a\}$; and $l(Human, w_1) = \emptyset$. In words, we have a model with three worlds, two of which contain one human, and one of which contains one non-human. The claim $(\exists x)\Box(Human(x))$ is true at every world of this model, meaning it is valid in this model. However, there are several ways to alter the model that will result in the falsity of the claim at

some world of the model. To illustrate this point, I shall introduce a modified version of M . Let M' be like M , except that R' is reflexive. This means that $M', w_1 \not\models (\exists x)\Box(Human(x))$. The falsity of $(\exists x)\Box(Human(x))$ at some world may also be brought about by constructing further versions of M where either the worlds of W are altered, the interpretation of the predicate *Human* is changed or new domain-members are introduced.

In short, a modal predicate logic translation of a metaphysical claim can only be true or false relative to a model that specifies the information from which this truth-value is derived. But this means that the very information that is at question (whether the actual world is such that there is at least one individual who is necessarily human) must be specified before a translation of this metaphysical claim can be assigned a truth-value! On reflection, this is as things should be: for a logic that could answer metaphysical questions, by presupposing certain metaphysical positions in its syntax or semantics, would lose a great deal of the neutrality and flexibility that logical systems are intended to instantiate.¹² It appears that the purpose of working with modal predicate logic translations of metaphysical modal claims is not to automatically glean the meanings and truth-values of metaphysical claims, but is instead the simplified syntax and semantics. For this reason, I will continue to consider modal predicate logic translations rather than the natural language modal claims themselves.

Nevertheless, the lesson to be learned is that the task of making sense of metaphysical modal claims is not completed merely by studying modal predicate logic. It therefore seems that, in order to determine the meanings and actual truth-values of metaphysical claims, we need to determine which models the actual world can be part of. The aim of the remainder of the thesis will thus be to clarify the modal predicate logic truth-conditions, with the hope of reaching a point at which it is transparent whether or not there are possible worlds in which the claim embedded under a modal operator holds. A preliminary step towards this aim involves establishing what exactly models and possible worlds *are*. This shall be considered in the next section.

0.2 Motivating a Reductive Account of Modal Terms

Before an attempt is made at seeking a suitable clarification of the truth-conditions for modal terms, the type of definition of ‘possible world’ we require should be established. In §2.1, the difference between reductivist and

¹²Cocchiarella (1975) p.13.

primitivist definitions of modal terms shall be explained. Two principles that appear to motivate a reductivist account of modal terms shall be discussed in §2.2, and in §2.3 the second principle shall be examined in more detail and argued to serve as good motivation for modal reductivism.

0.2.1 Reductive and Primitive Definitions

The truth-conditions for modal predicate logic claims that were given in §1.1 make use of the notion of *possible worlds*. Whilst the role of possible worlds within Kripkean semantics has been described (they are the points at which modal predicate logic *uffs* are assigned truth-values), a more robust definition has not been given. We might expect the provision of a more robust definition to clarify the sorts of models that the actual world may be part of.

When we ask how a theory defines the term ‘possible world’, we are enquiring into what Quine has called the *ideology* of that theory.¹³ Definitions of ‘possible world’, and hence the truth-conditions of modal claims, may be either *reductive* or *non-reductive*. To explain the difference between these views, it is worth repeating the truth conditions earlier given for modal predicate logic claims that feature the possibility operator, making the use of the term ‘possible world’ explicit:

$$M, \Gamma \Vdash_v \Diamond \phi \iff \text{there is some possible world } \Delta \in W : \Gamma R \Delta \text{ \& } M, \Delta \Vdash_v \phi$$

A *reductive* approach to modality is, roughly, one that gives a definition of ‘possible world’ (and, subsequently, the meaning of the modal operators) through the use of non-modal terms. The term ‘possible world’ is thus reduced to more logically basic non-modal terms, with the meanings of modal operators then being reduced to a definition in terms of possible worlds.¹⁴ A *primitive* approach to modality is one that only defines ‘possible world’ through making use of modal terms, thereby taking the meaning of the modal operators as logical primitives.

¹³Quine (1951), p.14.

¹⁴There is a second variant of reductivism, where ‘possible world’ is taken as an undefined primitive. Modal terms would then be said to be reducible to possible worlds, with no further analysis of the term ‘possible world’ being given. The trouble with this view is that it is difficult to argue that ‘possible world’ is a suitable primitive, given that it has a poor claim to being ‘immediately understandable’ (see §2.2). I shall therefore not consider this variant to be a viable option, and shall take it for granted that the expression ‘possible world’ has to be defined either through modal or non-modal terms.

To make this distinction more precise, let the following criteria hold: a reductive definition must be both *materially-adequate* and *non-circular*.¹⁵ A potential reduction is materially-adequate only if the left-hand side of the bi-conditional and the right-hand side receive the same truth-value. A proposed reduction is non-circular only if the right-side of the bi-conditional features neither terms from the proscribed class (i.e. modal terms) nor expressions which are themselves defined through the use of terms from the proscribed class.¹⁶ Having clarified what it means for an account of modality to be reductive, the remainder of §2 may be devoted to motivating a reductive account.

0.2.2 Two Principles Motivating Modal Reductivism

The motivations for modal reductivism will now be outlined. Having noted the instability associated with taking a term as a primitive, two principles that may be taken to motivate modal reductivism shall be considered. It will turn out that the second principle is a more promising way to argue for modal reductivism, whilst simultaneously being particularly difficult to rigorously define.

Taking a term to be primitive, and ruling out its being defined through an appeal to other terms, implies that we hold the term to be ‘immediately understandable’.¹⁷ Therefore, using a term as a primitive requires confidence that the vague hints we might drop about its intended interpretation (in lieu of a full definition) will combine with an intuitive reading of the term, in order to lead the reader to understand its meaning. It thus seems that there is an *instability* associated with taking a term as primitive that is lacking when it is defined. With a clear definition of a term, there is a way to guarantee that the (attentive) reader will grasp the intended and correct meaning of the term, provided the primitives used to define it are immediately understandable; yet there is no guarantee that a reader (attentive or otherwise) will greet an undefined primitive with immediate understanding.

An individual must obviously take some terms as primitives. However, this absence of a guarantee that the meaning of a primitive will be adequately conveyed results in the following two principles’ being compelling:

1. ‘*Picking Plausible Primitives*’: one should only commit oneself to

¹⁵Sider (2003), p.197.

¹⁶More precisely, we should say that no modal term is used to *directly or indirectly* define a term on the right-hand side (see §2.3).

¹⁷Tarski (1995), p.118.

primitives that seem likely to be ‘immediately understandable’ to all those who encounter them;

2. ‘*Limiting Primitives*’: one should commit oneself to as few primitives as possible in the process of giving adequate definitions.¹⁸

Both of these principles seem fairly uncontroversial. However, there may be doubt regarding their implications for modal discourse. The first principle will now be discussed briefly, as it will not be directly employed in arguments for modal reductivism.

To see that the first principle is uncontroversial, consider a scenario in which a term that is subject to much philosophical debate over its correct analysis is taken to be primitive. For example, suppose an ethicist wishes to give a reductive definition of the term ‘evil’. In order to achieve this aim, he takes the word ‘good’ as a primitive, and proposes as reductive truth-conditions: “x is evil’ is true if and only if x is not good’.¹⁹ This analysis is unlikely to strike many ethicists as an adequate definition of ‘evil’, given that their apparent puzzlement over the meaning of the word ‘good’ prevents its having a good claim to being immediately understandable to them.

Holding that the principle of picking plausible primitives is violated by treating modal terms as primitive remains a more controversial proposal. Of course, if one accepts the first principle, and one accepts that modal terms do not seem to be immediately understandable, then it follows that a reductive definition of modal terms should be sought. I am personally convinced of the idea that modal terms are not immediately understandable; otherwise, I would hardly have chosen to write on the topic of the meaning of modal expressions, unless I was content with an unusually short thesis. However, other people have informed me that modal terms strike them as good candidates for immediately understandable terms. Since debate over whether or not modal terms violate the first principle rests predominantly on subjective intuitions, I will not pursue this line of thought.

My emphasis will instead be on the second principle. To reveal how compelling the idea is that primitives should not be unnecessarily multiplied, consider the following scenario: Take a language that only contains the terms t' , t'' and t''' , with attempts’ being made to define such terms. Definition 1 takes all three terms to be primitives, whereas Definition 2

¹⁸A typical statement of this principle in the literature is: ‘According to one principle of simplicity, more primitive notions are worse than fewer; what primitive ideology we can eliminate, we should.’ (Wang (2013) p.538.)

¹⁹This would obviously exclude morally neutral actions, and so would be an unsatisfactory analysis on other grounds.

takes t' and t'' as primitives whilst reductively defining t''' through making use of the first two terms. It is difficult to imagine how someone could justify endorsing Definition 1 in such a scenario. For, given the uncertainty involved in conveying the meaning of a primitive, it seems to be preferable to minimise the number of such terms that one commits oneself to.

This should illustrate the plausibility of a principle that dictates that one should endorse as few primitives as one can get away with in giving adequate definitions of terms. Yet it is surprisingly difficult to clarify the limiting primitives principle in such a way that the idea behind it is captured. This shall be attempted in the following section, before the clarified principle is applied to the topic of modal terms.

0.2.3 Precisifying the Principle of Limiting Primitives

A precise definition of what it means for an individual to be *committed to a primitive* must be given, in order to elaborate the limiting primitives principle. Let T be the set of all the terms of a certain language, and define a binary relation \leq_S on T . This relation is intended to represent the means by which an individual S defines the terms of her language. In order to reflect important characteristics of the process of defining terms, \leq_S will be a *pre-order*. That is:

- \leq_S is *reflexive* ($t \leq_S t$).
- \leq_S is *transitive* (if $t \leq_S t'$ and $t' \leq_S t''$ then $t \leq_S t''$).

Anti-symmetry does not hold, since although we want to say that $t \leq_S t'$ and $t' \leq_S t$ if and only if $t =_S t'$, we would not want to say that $=_S$ is *identity*; rather, it is an *equivalence relation*, and an anti-symmetric partial order on these equivalence classes could then be defined. The motivation for denying that $=_S$ is identity derives from the fact that terms are not usually individuated entirely by means of the way in which they are defined, since the morphemes that compose them must also be taken into account. It should also be noted that $t <_S t'$ if and only if $t \leq_S t'$ and $t \neq_S t'$. We may then use the relation to capture several relevant notions:

1. S uses t' to define $t \iff t \leq_S t'$
2. S uses t' to directly define $t \iff t \leq_S t' \wedge \forall t''((t \leq_S t'' \wedge t'' \leq_S t') \rightarrow (t =_S t'' \vee t' =_S t''))$

3. S treats t, t' as synonyms $\iff t =_S t'$
4. S treats t, t' as independent terms $\iff \neg(t \leq_S t') \wedge \neg(t' \leq_S t)$
5. S gives a non-circular definition of t $\iff \forall t'(t <_S t' \rightarrow \neg(t' <_S t))$
6. S treats t as a primitive $\iff \neg \exists t' : t <_S t'$
7. S reductively defines t $\iff S$ gives a non-circular definition of t such that t is not treated as a primitive (and a bi-conditional featuring the terms t' that S uses to directly define t is materially-adequate).

It initially seems as if characterising these notions allows us to unproblematically define the set $P_{S,t}$ of all primitive terms to which an individual S 's definition of t commits him in a simple manner. As an intermediate step, I will attempt to define the set $R_{S,t}$ of terms *relevant* to the definition of t for S (this will be the set of terms that we search for primitives):

$$(First\ Attempt): R_{S,t} = \{t' \in T : t \leq_S t'\}$$

$$P_{S,t} = \{t' \in R_{S,t} : \neg \exists t''(t' <_S t'')\}$$

It might then appear that one could compare the cardinalities of the two sets of primitives that the rival definers S, S' commit themselves to with their definitions of t .²⁰

However, these definitions are too narrow to capture the intuitive idea behind the principle of limiting primitives. Firstly, consider these notions applied to the individuals *Prim* and *Red* with regards to their (respectively) primitivist and reductivist definitions of t . It is clear that $P_{Prim,t} = \{t\}$, hence $|P_{Prim,t}| = 1$; yet it is likely to be the case that $|P_{Red,t}| > 1$. Adopting this notion would result in the limiting primitives principle's recommending to take *every* term of T as a primitive ('*pan-primitivism*'). For it would then be guaranteed that only a single primitive would be used in giving a definition of each term, yet the reductivist may sometimes find herself in need of multiple primitives in order to give an adequate reductive definition of some term. This is precisely the opposite of what the principle of limiting primitives was intended to dictate.

²⁰For simplicity I have suggested that \leq_S is indexed to individuals, but a more accurate picture would be to say that it is indexed to world-time-individual triples. This would allow the same individual to consider multiple definitions of the same term, provided their considering each definition did not occur at the exact same time.

One might attempt to fix this idea of commitment by stipulating that it is only appropriate to compare the cardinalities of $P_{S,t}$, $P_{S',t}$ when neither S nor S' takes t itself as a primitive: in cases like the one earlier described, Prim automatically ‘loses’ to Red according to the principle of limiting primitives, and she may save her own account from embarrassment only by questioning the material-adequacy of Red’s definition. Yet this modified notion is still far-removed from the idea behind the limiting primitives principle. For imagine that Prim has summoned the energy to give a reductive definition of t by means of two terms t', t'' , but that she has refrained from considering how to define any other term of T (and is thus assumed to take them as primitives). Suppose Red has reductively defined t in such a way that it is incorporated into a complex web of one hundred inter-defined terms of T that she has been considering, with ten of these terms taken as primitives. The principle of limiting primitives should surely favour Red’s approach: for, out of those one hundred terms, Prim ends up with one hundred primitives. Yet since those one hundred primitives are incomparable to t according to the \leq_{Prim} relation, they are not taken into account. The principle of limiting primitives would therefore seem to penalise prolific theorists, and would deem the ideal situation to be one in which high numbers of primitives are employed to ensure that terms are independent.

A more effective means of comparing primitives must therefore be sought. The hope would be to find a way to broaden the notion of comparison such that any term *either* individual S or S' uses to define t becomes relevant in determining *both* $P_{S,t}$ and $P_{S',t}$. Having separated the sets $R_{S,t}$ and $P_{S,t}$, we may seek the desired effect by experimenting with the membership criteria for the set of relevant terms alone. Let us therefore say that, given a particular rival S' :

$$(Second\ Attempt): R_{S,t} = \{t' \in T : (t \leq_S t') \vee (\exists t'' : t \leq_{S'} t'' \wedge t'' \leq_S t')\}$$

This modification helps with the two earlier described problematic cases: firstly, the primitives that definitions of t commit their theorists to may be compared even if one individual takes t itself as a primitive. That is, $P_{Prim,t} = \{t \cup \{t' \in T : (\exists t'' : t \leq_{Red} t'' \wedge t'' \leq_{Prim} t') \wedge (\neg \exists t'' : t' <_{Prim} t'')\}\}$; hence it need no longer be the case that $|P_{Prim,t}| = 1$, and the principle of limiting primitives will no longer recommend pan-primitivism. Secondly, Prim will be committed to any primitives that she employs in defining any term that Red uses to define t . This precludes situations where Prim may escape commitment to primitives simply by virtue of being a lazy theorist and treating the majority of terms as independent.

However, there is one further situation that this improved attempt continues to neglect. Suppose Red uses t' to define t , but Prim treats t and t' as independent. According to the modified definition, Prim is now committed to any primitives that she associates with t' by virtue of Red's judgement of the relevance of t' . This is as it should be. But suppose Prim uses the primitives t'' and t''' to define t' , whereas Red considers the three terms t'', t''' and t' to be independent at the same time as treating the two former terms as primitives. Now Red will avoid commitment to the primitives t'' and t''' . This is the case because Prim has not treated t' as connected to t , and hence the terms she uses to define t' will not be captured by the modified definition. So now we seem to disadvantage the theorist who postulates many independent terms! Rather than attempting to implement further specific criteria for set-inclusion, it seems sensible to switch to the following formulation:

$$(Final\ Version): R_{S,t} = \{t' \in T : (t \leq_S t') \vee (\exists t'' \in R_{S',t} \wedge t'' \leq_S t')\}$$

In essence, this involves importing a recursive definition of $R_{S,t}$ into the membership criteria for $R_{S,t}$ itself. For our 'base case', it is specified that all terms S uses to define t are relevant; but a given rival S' will end up with a set $R_{S',t}$ that also includes all terms S' uses to define every term in $R_{S,t}$ due to the new disjunct in the inclusion criteria; and then $R_{S,t}$ will be required to contain all terms S uses to define any new term in $R_{S',t}$, and so on. It is plausible to assume that genuine definitions of any term t must always be finite, so a stipulation can be introduced that rules out infinite chains of associated terms:

$$1. S \text{ uses } t' \text{ to define } t \iff t \leq_S t' \wedge \exists t'' (t' \leq_S t'' \wedge \neg \exists t''' (t'' <_S t'''))$$

Of course, $R_{S,t}$ may still have a very large cardinality if either S or his rival S' turn out to be prolific definers with webs of related terms. Yet it seems safe to assume that, in general, an excessively large number of terms will not end up being relevant to a single term t . For example, it is unlikely that any S, S' considering the term 'possible' will make use of the term 'brisket'. This final version of the definition of the terms that are relevant to a term being defined, relative to at most two individuals, shall therefore be employed. I shall now repeat the definition of the set of primitives $P_{S,t}$ that S 's definition of t commits her to, before taking stock:

$$P_{S,t} = \{t' \in R_{S,t} : \neg \exists t''(t' <_S t'')\}$$

It should be noted that we have ended up with *two* notions of commitment to primitives: the first *non-comparative* notion arises when no rival definer is chosen, and the second *comparative* notion occurs when a particular rival is selected (and the primitives S is committed to will then depend on that rival).

How should we employ these notions in clarifying the principle of limiting primitives? First, suppose we take the principle to advocate the minimising of primitives that one commits oneself to in the non-comparative sense. The trouble is that the principle of limiting primitives then seems to once more advocate pan-primitivism, since a primitivist about t only commits himself (non-comparatively) to a single primitive. On the other hand, suppose we take the principle to advocate the minimising of primitives that one commits oneself to in the comparative sense. The problem is that there is an infinite number of rival definitions, so we then seem to require S to count the number of primitives she (comparatively) commits herself to relative to every possible rival theory. Since an individual might not have a full range of rival definitions in mind when developing his definition of t , this is a difficult condition to reliably pursue.

It seems clear that the use of the term ‘commitment’ in the principle of limiting primitives should be interpreted in the comparative sense; for the problem with such an interpretation concerns how to pursue the recommendations of the principle, rather than whether it yields the correct verdict. A means of reacting to the epistemic uncertainty associated with limiting one’s commitments relative to infinite hypothetical rivals shall now be sought.

Suppose we take the principle of limiting primitives to advocate the minimisation of the primitives one commits oneself to in the comparative sense, but then describe a means to increase the *likelihood* that one will commit oneself to fewer primitives than any rival. Of course, the fact that S will never compare his definition of t to every other conceivable definition means that his assessment of likely success regarding the limiting primitives principle is always defeasible. Yet this is seemingly the only approach that has a chance of describing a means to implement the principle.

Consider the following as a means to increase the chances of one’s limiting one’s primitives: *If one can reductively define a term t , and one’s proposed reduction includes at least one term t' that is difficult to reductively define, then reductively define t and take t' as a primitive.* A term t' might be thought to be *difficult to reductively define* if no combination of terms can be found that provide a reductive definition, or if t' appears to have a good

claim to being a plausible primitive.²¹ The best evidence for t' 's being difficult to reductively define will be if most other definitions of t' take it as a primitive, provided we assume that everyone is tacitly operating under the principle of limiting primitives.

The justification for this means of increasing the likelihood of minimising one's primitives is explained by the following two considerations: firstly, the principle of limiting primitives clearly advises a policy of giving reductive definitions where one can get them. For if one wishes to define the terms $t1 \dots tn$, then an individual who takes all the terms as primitive can be guaranteed to commit herself to at least n primitives when she comes to compare her definition to a rival one; whereas the individual who reductively defines at least one term of $t1 \dots tn$ through making use of some of the other terms opens up the possibility of committing himself to fewer than n primitives when a comparison occurs. Of course, there may be cases where t can be reductively defined by means of t' and t' can be reductively defined by means of t . Such cases explain the presence of the second aspect: if one suspects that one is using a primitive t' in one's reductive definition of t that is not defined reductively by any rival, then the likelihood that one's rival will be committed to t' once comparison occurs increases. Then, if any rival takes t as a primitive and also cannot avoid taking t' as a primitive, the individual who reductively defines t in terms of the primitive t' would increase the chances of committing himself to at least one fewer primitive than the rival; and if any rival reductively defines t in some different way but also ends up taking t' as a primitive, then the individual who reductively defines t in terms of the primitive t' will increase the chances of committing himself to no more primitives than the rival in defining t .

From the principle of limiting primitives and the above condition that clarifies a means to increase the likelihood of one's minimising one's primitives, the following implications for modal reductivism result: *if modal expressions can²² be given a reductive definition, and this reductive definition*

²¹In §2.2, the principle of picking plausible primitives was said to be uncontroversial but ultimately a poor motivation for modal reductivism. However, there is no reason to refrain from utilising it as a part of a motivation for modal reductivism, especially when it is applied not to modal terms themselves but to the terms used to define the modal terms. One must simply bear in mind that the application of this principle is unlikely to be decisive: for whenever S states that t' is a plausible primitive, S' may come along and say that, according to her own intuitions, t' is a ludicrous primitive. The subjective element to this principle therefore prevents its taking more than a minor role in a motivation for reductivism.

²²It is unfortunate but unavoidable that a modal term is being used in the elaboration of this principle, when the question we are currently concerned with is the analysis of

uses non-modal primitive terms that are difficult to reductively define, then modal reductivism becomes a compelling position.

In this section, it has been argued that we should seek a reductive definition of ‘possible world’ that increases the likelihood of our (comparatively) committing ourselves to fewer primitives than any rivals. Having clarified the sort of definition that we should seek, we may proceed with our search.

0.3 The Type of Reductive Account to Pursue

§3 shall further narrow down the type of reductive definition we should seek. §3.1 gives a fairly exhaustive list of the different definitions of the term ‘possible world’ that are prevalent in the literature. Then, in §3.2, it shall be argued that linguistic ersatzism is the only type of account that possesses the potential to yield a genuinely reductive definition.

0.3.1 Different Ways of Defining ‘Possible World’

Before an attempt at giving a suitable reductive account of modal claims is made, the range of options for a definition of ‘possible world’ shall be described. These options consist of *modal realism* and a variety of *ersatzisms*.

Lewis documents the different ways of construing possible worlds in his classic work ‘On the Plurality of Worlds’. Lewis himself is a *modal realist*, holding that ‘Our world is but one world among many’.²³ Modal realists consider the possible worlds alluded to on the right-hand side of the truth-conditions for modal predicate logic claims to be alike in kind to the ‘actual’ (from our perspective) world. However, each world is spatiotemporally inaccessible from all other worlds.²⁴ For Lewis, ‘A world is the mereological sum of all the possible individuals that are parts of it, and so are worldmates of one another’, where two things are ‘worldmates’ if they are spatiotemporally related.²⁵

Modal realism is opposed to the alternative view Lewis describes, *modal ersatzism*. Modal ersatzers hold that ‘instead of an incredible plurality of concrete worlds, we can have one world only, and countless abstract entities representing ways that this world might have been’.²⁶ These abstract entities

modal terms. I hope that my giving a successful reductive definition will be sufficient to show that a reductive definition *can* be given.

²³Lewis (1986) p.2.

²⁴*Ibid.* p.2.

²⁵*Ibid.* p.71.

²⁶*Ibid.* p.136.

are, importantly, part of the actual world.²⁷

This leads to three requirements for a view to be ersatzist: the worlds must be *actual*, *abstract* and connected to ways the world might have been through *representing* these ways. Modal realism, in contrast, holds possible worlds to be *non-actual*, *non-abstract* and connected to ways the world might have been through *instantiating* these ways.

Ersatzism comes in several varieties, according to the abstract structures chosen to represent other ways the world might have been. Lycan lists the forms of ersatzism that have been proposed by various philosophers, which shall now be summarised.²⁸

The first option is *linguistic ersatzism*, which holds that possible worlds are sets of sentences. Both Carnap and Hintikka have at some point endorsed a form of linguistic ersatzism. Hintikka construed his ersatz worlds as non-maximal sets of atomic and non-atomic sentences,²⁹ whereas Carnap held them to be maximal sets of atomic sentences.³⁰ Neither view requires the language out of which these worlds are constructed to be a natural language in a literal sense (finite, learnable, etc.);³¹ it need only be a language in the sense of having a syntax and semantics. Since possible worlds are constructed from sets and sentences, they are clearly *abstract*. Provided the sentences used are not defined as the sentences ‘possible’ relative to a given language, possible worlds will also be constructed only from *actual* entities.³² Finally, a possible world *represents* a state as obtaining on the basis of the inclusion of a sentence that describes that state in the set that composes the relevant possible world.

A second ersatz approach is *propositional ersatzism*, whereby worlds are constructed out of language-independent propositions. Adams endorsed this form of ersatzism, holding that a ‘world-story is a maximal-consistent set of propositions’.³³ On this ersatz view, possible worlds are *actual* and *abstract* in the sense of being composed only of sets and propositions, both of which are construed as actual, abstract entities. In addition, a possible world *represents* a state of affairs by including a proposition that expresses the obtaining of that state.

A third version of the ersatzist view is *combinatorial ersatzism*, which

²⁷Lycan (1994), p.15.

²⁸*Ibid.* pp.45-6.

²⁹Hintikka (1969) pp.71-86

³⁰Carnap (1988).

³¹Lewis (1986) p.144.

³²See §7 for a discussion of the linguistic ersatzist’s definition of ‘sentence’.

³³Adams (1974) p.225.

uses re-combinations of the atomic aspects that compose the actual world in defining ‘possible world’. Originally raised by Quine, Cresswell suggests a version whereby we consider the set B of all space-time points.³⁴ A ‘possible world’ Γ is then a subset of B , including only the points *occupied* at Γ . The actual world is represented by a subset of B including all and only the actually occupied space-time points, and every other subset of B counts as a non-actual world. Castaneda describes a variant of this view, holding that properties should be considered the ‘ultimate components of the world’.³⁵ There are various operators that collate properties into those instantiated by each individual.³⁶ A ‘possible world’ is a set of individuals, with each individual being a set of properties; and non-actual individuals may be represented by applying an operator to yield a set of properties that is not instantiated by any actual individual.³⁷ Although the atoms used to construct possible worlds may be held to be space-time points, properties or perhaps other structures entirely, all combinatorial ersatz views share the ideas that: the atoms that compose possible worlds are aspects of the *actual world*; *abstract* set-theoretic entities that are isomorphic to alternate arrangements of the atoms are to function as possible worlds; and a possible world *represents* a state of affairs as obtaining if the arrangement of atoms that it is isomorphic to determines that state of affairs.

This is a fairly exhaustive list of the available ersatz accounts. Although other varieties of ersatzism are occasionally mentioned in the literature, those that have been omitted from the current discussion will have been left out for one of the following two reasons:

1. No one has developed such a variant in detail, and so the features of a fully-elaborated account cannot be assessed;³⁸
2. It is a form of what Lewis has called ‘magical ersatzism’, which takes possible worlds to be structures that simply represent due to their nature and ‘there’s nothing to be said about how they do it’.³⁹

My complaint about magical ersatzism is that it is closely associated with the second variant of reductivism, described in the second footnote of §2.1,

³⁴Cresswell (1972) p.6.

³⁵Castaneda (1972) p.50

³⁶*Ibid.* p.51

³⁷*Ibid.* p.55

³⁸For example, Lycan’s idea (in Lycan (1994) p.46) of taking possible worlds to be ‘mental entities’ of some form has not received full consideration as of yet, and was omitted from the current discussion on these grounds.

³⁹Lewis (1986) p.141.

which takes the term ‘possible world’ as an undefined primitive. The doubts that I described about this reductive approach thus generalise to variants of magical ersatzism, which is why such views have not been included as viable ersatzist accounts in this section.⁴⁰

Finally, the space of worlds generated by a particular definition of ‘possible world’ may be categorised depending on the features it instantiates.⁴¹ One might define ‘possible world’ in such a way that the space of worlds generated is well-behaved, in the sense that the worlds that exist and fail to exist do so according to some underlying principle. Call such a space of worlds ‘*principled*’. In particular, one may want the space of worlds to obey a principle that ensures the space matches some pre-theoretical notion of the totality of possibilities. Alternatively, if one defines ‘possible world’ so that its extension merely includes a brute pattern of worlds obeying no such principle, then call the space of worlds ‘*unprincipled*’. If a space of (real or ersatz) worlds includes intuitive impossibilities and excludes intuitive possibilities, I shall refer to such a space of worlds as ‘*chaotic*’. Unprincipled spaces, then, have a marked propensity to be chaotic; whereas principled spaces may be devoid of chaos if the principle underlying their construction effectively causes the space to include all and only the intuitive possibilities.⁴²

At this point it should also be remarked that there are degrees of chaos. At the most extreme end, a space of worlds may be chaotic if there are worlds at which logical contradictions are true. Such a space of worlds would clearly fail to function as a materially-adequate reductive analysis of modal expressions, since it is generally accepted that if anything is impossible, then the truth of logical contradictions surely is. Then there are lesser degrees of chaos, whereby logical contradictions are successfully excluded from the space of worlds but the intuitive truth-values of other modal claims may fail to be reflected.⁴³ Distinguishing between degrees of chaos will therefore be

⁴⁰For example, Lycan’s elaboration (Lycan (1994) p.46) of Stalnaker’s view that possible worlds are to be construed as ‘ways things might have been’, which cannot be ‘reduced to items of any more familiar kind’, has been omitted from the current list due to its plausibly counting as a magical ersatzer view.

⁴¹When I use the expression ‘space of (possible) worlds’, I of course mean ‘space’ in some metaphorical sense: on the modal realist’s view, the spatiotemporal dislocation that obtains between worlds means that the pluriverse itself is not a ‘space’ in any literal sense; and on the ersatz picture, worlds are abstract objects, so are not spatiotemporally located at all.

⁴²Note that some principled spaces of worlds might turn out to be chaotic (i.e. if the principle that is used to generate them is ineffective). Similarly, an unprincipled space of worlds may turn out to be non-chaotic, through a massive stroke of good fortune.

⁴³Of course, it is often difficult to tell how much uniformity there is over the ‘intuitive’

important, because a space of worlds that might be argued to display some degree of chaos can more plausibly be shown to yield a materially-adequate bi-conditional (through questioning whether the modal facts represented in the space of worlds really do violate modal intuitions) than a space of worlds that displays total chaos.

In this section, a full range of ways of defining ‘possible world’ has been described, and a means to categorise the definitions according to the type of space of worlds they generate has been given. These matters have been discussed in preparation for an endorsement of linguistic ersatzism as the only account with the potential to be reductive.

0.3.2 Linguistic Ersatzism as a Reductive Account

It shall now be explained why many philosophers are dubious about the potential for a reductive ersatz account of modal terms. It will then be argued that linguistic ersatzism actually exemplifies the greatest potential for a reductive account of modal expressions.

There is a pervasive view, which may be traced back to Lewis, that only modal realism has the potential to yield genuinely reductive accounts of modal terms. This view is grounded in the perception that the modal realist ‘explicates modality just in terms of big, lumpy physical objects – no modal primitives, Lewis claims’.⁴⁴ However, I believe that Lewis ends up making use of tacit modal primitives in defining the term ‘possible world’, due to his attempts at generating a principled space of worlds that matches our pre-theoretical notions of possibility and impossibility. In short, Lewis’s definition of a ‘world’ as a mereological sum of spatiotemporally related individuals does not say enough to ensure that there is a possible world for each intuitive possibility. For example, it might so happen that there is no mereological sum in which horned horses exist. If we maintain that it is possible for horses to have horns, then the relevant bi-conditional has a false right-hand side and a true left-hand side, meaning that material-adequacy is lost. Lewis is aware of this, and attempts to generate a principled space of worlds through employing a principle of *recombination*. This principle holds that ‘anything can coexist with anything else, at least provided they occupy distinct spatiotemporal positions. Likewise, anything can fail to coexist with anything else.’⁴⁵ Hence, it is guaranteed that there is a world in which

truth-values of modal claims (though there can be some confidence that there is great uniformity in the judgement that the truth of logical contradictions is not possible)

⁴⁴Lycan (1994) p.38.

⁴⁵Lewis (1986) p.88.

horses coexist with appropriately placed horns. However, it should be clear that the quantifiers in this principle must range over non-actual individuals if the principle is to generate the principled space of worlds that Lewis wants: for Lewis believes that ‘alien’ individuals, composed of parts that are not duplicates of any parts of actual individuals, are possible.⁴⁶ If the principle is to deliver these alien individuals, it must range over every alien property in the first place. Yet it seems that alien individuals and properties would only be available to the quantifiers on the grounds that they are (in Lewis’s eyes) *possible*. I therefore accept the argument that the principle of recombination uses a term the sense of which can only be conveyed through modal terms (specifically, ‘anything’ is tacitly interpreted to mean ‘anything *possible*’), and hence fails the non-circularity condition.⁴⁷ Moreover, if a modal realist were to eliminate principles that contain modal terms from their definition of ‘world’, then the resulting unprincipled space would be likely to be chaotic (e.g. due to the omission of worlds in which there are horned horses) and hence fail the material-adequacy condition. Therefore, modal realism arguably fails to result in a genuinely reductive account of modal terms, since it violates either the non-circularity or material-adequacy conditions.

I now aim to show Lewis’s claim that no ersatz account can be reductive to be similarly debatable. The potential for reductive ersatz accounts shall be considered, and it will be argued that linguistic ersatzism seems to present a promising reductive approach.

It is first worth examining Lewis’s reasons for holding that no ersatz view can be reductive in more detail. Lewis’s central argument is that the only way to exclude contradictory states of affairs from being represented in the ersatz worlds without using an explicit modal term would be to employ some notion of *consistency*. However, he holds that the only plausible definition of ‘consistency’ that successfully rules out intuitively impossible worlds ‘is *prima facie* a modal distinction: a set of sentences is consistent iff those sentences, as interpreted, *could* all be true together’.⁴⁸ Likewise for the definition of ‘consistency’ for propositions, properties or occupied space-time points. Of course, the definition Lewis describes does make use of a modal term, meaning that any ersatz view that defines a ‘possible world’ as a set of Lewisian-consistent something-or-other would, on analysis, be revealed to fail the non-circularity condition.

⁴⁶Lewis (1986) p.91.

⁴⁷For further discussion on the circularity of Lewis’s putative reductive definition, see Divers (2002) p.121; and Lycan (1994) p.38 and pp.87-9.

⁴⁸Lewis (1986) p.151.

However, an alternative notion of consistency may be defined that avoids violating the non-circularity condition. This alternative notion of consistency is a *syntactic form* of consistency that will be argued in §4.1 to avoid invoking any modal primitives, thereby being non-circular. Such syntactic consistency may be defined as a property of sets of sentences; however, it cannot be defined as a property of non-linguistic entities such as mereological sums of individuals, sets of atomic simples or sets of propositions. This prevents it from being similarly used in the definition of ‘possible world’ for modal realists, combinatorial ersatzers or propositional ersatzers. Therefore, whilst Lewis’s argument against the potential for reductive ersatz accounts is correct with regards to propositional and combinatorial ersatzisms, linguistic ersatzism has not yet been ruled out as a viable reductive account. Moreover, given the arguable failure of modal realist reductive accounts, *only* linguistic ersatzism has the potential to give a successful reductive account of modal terms. Linguistic ersatzism will thus be focused on hereafter.

Whilst it has just been argued that linguistic ersatzism instantiates the greatest potential for a successful reductive account, it has not yet been determined whether we should adopt a reductive, linguistic ersatz account. Recall that, in §2.3, it was concluded that if modal expressions can be given a reductive definition that is associated with non-modal primitives that the rival definers are likely to be committed to anyway once the definitions are compared, then we should be modal reductivists. It was also said that a rival definer is *likely* to be committed to a primitive term *t* upon comparison of definitions if *t* is difficult to reductively define. Finally, *t* is *difficult to reductively define* if no combination of terms that allow a successful reductive definition are forthcoming, or if *t* appears to have a good claim to being a plausible primitive. We must therefore check whether the linguistic ersatzers’ primitives, *set* and *sentence*, are difficult to reductively define. Now, Lewis himself notes that linguistic ersatzism ‘fulfils its promise of safe and sane ontology’ by relying on ‘ontology that most philosophers are committed to in any case’.⁴⁹ Let us assume that one is committed to incorporating some *x* into one’s ontology just in case a term used to refer to *x* is endorsed as a primitive. Lewis’s comment may then be read as a concession that most philosophers accept the terms ‘set’ and ‘sentence’ as primitives. It is certainly the case that the term ‘set’ is taken as a primitive within Zermelo-Fraenkel set theory.⁵⁰ Furthermore, within linguistics, there is a fairly tight circle of inter-definable terms (such as ‘sentence’, ‘string’, ‘utter-

⁴⁹Lewis (1986) p.143.

⁵⁰Leung and Chen (1992) p.vii.

ance', 'grammatical', 'language', etc.), at least one of which must be taken as primitive and used to reductively define the others. Since widespread acceptance of a primitive term was said to be good evidence of its being difficult to reductively define, we may conclude that the terms 'set' and 'sentence' are difficult to reductively define.⁵¹ All of this indicates that rival definers are likely to be committed to the primitives endorsed by the linguistic ersatz. Hence adopting a reductive linguistic ersatz account increases the likelihood that the number of primitives one (comparatively) commits oneself to will be less than that of a rival. This completes the motivation to adopt a successful linguistic ersatz account, should one be found.

0.4 A Reductive Linguistic Ersatz Account

In §4.1, 'consistency' and 'maximality' shall be defined in such a way that using them to explicate the term 'possible world' meets the non-circularity condition. It will be conceded in §4.2 that the material-adequacy of the proposed account is more difficult to defend, in light of an objection from Lewis. In §4.3 the proposed account will be modified to include assumption-relative accessibility relations and assumption-relative modal operators. §4.4 will then argue that this modified account is materially-adequate in addition to being non-circular.

0.4.1 A Non-circular Definition of 'Possible World'

As mentioned in §1.1, modal predicate logic's basis in classical logic means that it assumes both the *law of non-contradiction* and the *law of excluded middle*.⁵² This means that the set of formulas true at a possible world must be both *consistent* (no formula P is both true and false at a single world, from the law of non-contradiction) and *maximal* (every formula P of the relevant language is either true or false at each world, after the law of excluded middle). Given that these two conditions must hold for the set of *wffs* true at each world, it seems that they may be of use in characterising possible worlds. Having defined consistency and maximality more rigorously, the way in which these notions may be used in giving a non-circular definition of the term 'possible world' shall be elaborated.

Where S is a set of *wffs* formed using the predicates, variables, connectives and quantifiers of the *non-modal* component of a language of modal

⁵¹In those cases where 'sentence' is reductively defined, the definer will still commit himself to at least one related primitive (e.g. 'language').

⁵²Kleene (2002), p.8.

predicate logic MPL' ,⁵³ and L' is a system of modal logic, S is L' -inconsistent iff there are $\phi_1 \dots \phi_n \in S$ such that $\vdash_{L'} \neg(\phi_1 \wedge \dots \wedge \phi_n)$. S is L' -consistent iff S is not L' -inconsistent. In words, a set of sentences S is *consistent* for a language and system of modal predicate logic if and only if there is no finite subset of S for which the negation of the conjunction of its members is a theorem of the relevant system of logic.

A set S of wffs is said to be *maximal* iff, for every wff ϕ of $MPL'_{-\Box, -\Diamond}$, either $\phi \in S$ or $\neg\phi \in S$.

A set S of wff is *maximal-consistent* iff, unsurprisingly, S is both consistent and maximal. It is provable that every consistent set can be extended to a maximal-consistent set.⁵⁴ This is achieved by showing that, given a consistent and non-maximal set S , for every wff P either P or $\neg P$ can be added to S , and a consistent set will result. For suppose $S \cup P$ resulted in an inconsistent set, and $S \cup \neg P$ also resulted in an inconsistent set. From the definition of consistency, this would mean that there would be some $\phi_1 \dots \phi_n \in S$ such that (i) $\vdash_{L'} \neg(\phi_1 \wedge \dots \wedge \phi_n \wedge P)$ and (ii) $\vdash_{L'} \neg(\phi_1 \wedge \dots \wedge \phi_n \wedge \neg P)$. Since it can never be the case that $\vdash_{L'} \neg(P)$ and $\vdash_{L'} \neg(\neg P)$ (by the law of non-contradiction), (i) and (ii) can be true only when $\vdash_{L'} \neg(\phi_1 \wedge \dots \wedge \phi_n)$. But this would mean that S itself would be inconsistent, contra the original assumption. We take the union of all these additions, which is guaranteed to exist by Zorn's lemma.⁵⁵ It is then easy to show that the union is maximal-consistent. Furthermore, we require each maximal-consistent set S to have the ' \forall -property'.⁵⁶ It is thus clear that, for every set S that is consistent but non-maximal, there will be a maximal-consistent extension.

A maximal-consistent set of wff S may be said to be *connected* with a world Γ iff, if a wff ϕ is true at Γ , then $\phi \in S$; and, conversely, if $\phi \in S$ then ϕ is true at Γ . Since it can be proven that every consistent set can be extended to a maximal-consistent set, this means that every consistent set

⁵³We could of course define maximal-consistency with regards to the modal component of MPL' as well. However, the account that I will go on to develop does not require the inclusion of modal expressions in the maximal-consistent sets that worlds are construed as in order to be coherent. Furthermore, the reductive aspect of the account is most convincingly implemented by eliminating modal terms entirely from the maximal-consistent sets invoked on the right-hand side of the bi-conditional.

⁵⁴Cresswell and Hughes (2012), p.115-6.

⁵⁵See Enderton (2001), p.7.

⁵⁶This is to rule out situations in which $\neg\forall x\phi x \in S$ but $\phi y \in S$ for all variables y . In such a scenario, every finite subset of S is L' -consistent, but there is no witness to $\neg\forall x\phi x$, and no witness can be added without rendering S inconsistent. In short, if S is maximal-consistent and has the \forall -property, then if $\forall x\phi \notin S$, there must be a witness y such that ϕ_y^x (ϕ with all occurrences of x replaced with occurrences of y) $\notin S$. For more details, see Cresswell and Hughes (2012), p.257.

can be extended to a set that is connected with some world.

The thought might then arise that a possible world just *is* a maximal-consistent set of *wff*. Without good reason to refrain from considering the connection relation to be identity (that is, S is connected with Γ iff $S = \Gamma$), it seems that we might as well do so. The motivation for this approach would be its plausibility as a non-circular definition of the term ‘possible world’. Consider the version of the bi-conditional that will result from explicating the definition of a ‘possible world’ that has been given:

- $M, \Gamma \Vdash_v \Diamond \phi \iff$ there is some *maximal-consistent set* $\Delta \in W : \Gamma R \Delta$ and $\Delta \Vdash_v \phi$
- Furthermore, if ϕ is free of modal operators, then $\Delta \Vdash_v \phi \iff \phi \in \Delta$.

It will now be argued that these new truth-conditions are non-circular. To be non-circular, it was firstly said that modal terms cannot appear on the right-hand side of the bi-conditional (except possibly in ϕ). This condition has clearly been met. Secondly, vocabulary which can only be adequately understood by using modal terms is not permitted on the right side. The definitions of maximality and consistency do not make use of such vocabulary. Therefore, these truth-conditions are non-circular. With this definition in mind, we can continue to use the term ‘possible world’, despite the presence of the modal term ‘possible’; for it has been shown that the term ‘possible world’ is non-circular under the proposed analysis.

0.4.2 Lewis’s Material-Adequacy Objection

It will now be conceded that the most significant argument against the proposed linguistic ersatz account is one that questions its material-adequacy. A modification to the account, brought about through re-conceiving of the accessibility relation, will be argued to ensure material-adequacy.

Lewis opposes linguistic ersatz accounts that make use of the non-circular, syntactic form of consistency in their putatively reductive definition of the term ‘possible world’. His objection is that they generate a chaotic space of worlds. This is the case because this syntactic definition of ‘consistency’ only prevents the affirmation and denial of the very same formula, which means that it yields ‘allegedly consistent ersatz worlds according to which there are married bachelors, numbers with more than one successor, and suchlike impossibilities’.⁵⁷

⁵⁷Lewis (1986) p.153.

To see Lewis's point, let the truth-conditions for the sentence 'It is possible that there is at least one married bachelor' be translated into a language MPL' as follows:

$$M, \Gamma \Vdash_v \Diamond(\exists x)(Mx \wedge Bx) \iff \exists \Delta \in W : \Gamma R \Delta \text{ and } M, \Delta \Vdash_v (\exists x)(Mx \wedge Bx)$$

According to the current definition of 'possible world', the right-hand side of the above bi-conditional is *true*: for, as Lewis points out, there is no syntactic inconsistency in the *wff* $(\exists x)(Mx \wedge Bx)$. This is the case because M and B are two unrelated predicates in the modal predicate logic language. Yet Lewis claims that $(\exists x)(Mx \wedge Bx)$, when interpreted as the English claim 'it is possible that there is at least one married bachelor', is intuitively *false*. The fact that this sentence is intuitively false is due to 'the logical connections that hold between interpreted non-logical terms'.⁵⁸

There are two choices about how to construe this data: firstly, one might disagree with Lewis's claim that the 'allegedly consistent worlds' do instantiate intuitive 'impossibilities'. Secondly, one might agree with his claim. Both strategies seem to lead to problems for the proposed linguistic ersatzer account.

Firstly, consider someone who claims, contra Lewis, that married bachelors are possible. It is unlikely that such a person reached this view on the basis of her own pre-theoretical intuitions about the truth-value of the claim 'It is possible for there to be married bachelors', or on the basis of her pre-theoretical presumptions about the intuitions the majority of people will have about this claim. This is the case because it is difficult to deny that the natural response to an individual who said 'Did you know, it's possible for there to be married bachelors?' would be to assume that he had misunderstood the terms or was in the process of telling a joke. The person who disagrees with Lewis is more likely to believe that married bachelors are possible on the basis of the fact that there are syntactically consistent linguistic ersatzer worlds in which there are married bachelors. In short, she would seem to have such a commitment to the proposed analysis of modal claims that she would believe that, if a world exists in which the proposition that P holds, then $\Diamond P$ is true; and if no world exists in which the proposition that P holds, then $\Diamond P$ is false. She will therefore respond to any accusations of material-inadequacy by arguing that, contra the standard intuitions, the truth-value of the disputed modal claim does in fact match the truth-value

⁵⁸Ballarin (2005) p.279.

of the putative reduction on the right-hand side.

However, it should be recalled that the aim of a reductive definition is to clarify the meaning of the *analysandum*. Disregarding the truth-values that a high proportion of people intuitively attribute to the left-hand side of the relevant bi-conditional is a troubling way to proceed. For if one could stipulate that the truth-values of claims on the right-hand side of the bi-conditional determine those on the left, then material-adequacy will be guaranteed no matter how tenuous the connection between the *analysandum* and the *analysans*. To illustrate this concern, Shalkowski (1994) writes that ‘Instead of thinking that $[W]$ is a set of worlds, think of it as the set of pencils in my drawer, with ... R as a relation among these pencils. Clearly, anyone proposing truth about all these pencils as necessary truth and truth about at least one pencil as possible truth would be guilty of co-opting the terms ‘necessary’ and ‘possible’ and giving them stipulative definitions’.⁵⁹ In short, this first strategy is susceptible to an *arbitrariness* objection: that is, if the intuitive meaning and truth-value of the *analysandum* is ignored in order to maintain material-adequacy, then it is not clear why one should attempt to reductively define modal terms by means of maximal-consistent sets rather than truths about the pencils in Shalkowski’s drawer. Therefore, priority must be given to the truth-values of the claims on the left side of the bi-conditional, as far as we can discern them.

What, then, of the second strategy? If one agrees with Lewis that it is impossible for there to be married bachelors, then the material-adequacy of any account that attempts to reductively define ‘possible world’ as ‘maximal (syntactically)-consistent set of sentences’ is undermined. For the truth-values of the left-hand and right-hand side of the bi-conditional will differ in some cases. On this perspective, the space of worlds generated by the current account will display some degree of chaos less extreme than total chaos: for although we might take cheer from the fact that logical contradictions are still correctly ruled to be impossible (preventing total chaos), it can be demonstrated that a class of impossibilities marginally less offensive than logical contradictions are represented in the space of worlds.

The proponent of the current account does not look to be in a good position: she is barred from claiming that it is in fact true that it is possible that there are married bachelors, since she would seemingly be making this claim due to her prioritising the implications of her theoretical commitments, thus violating the non-arbitrariness condition. The proponent of the current account also encounters problems when she concedes that the

⁵⁹Shalkowski (1994) p.279.

proposition expressed by the sentence ‘It is possible for there to be married bachelors’ is intuitively false, as the proposed reductive definition of modal terms therefore lacks material-adequacy.

Fortunately, I believe that there is a third response to Lewis’s objection, which avoids dooming the current account to fail either the non-arbitrariness or material-adequacy conditions.

0.4.3 Relative Accessibility Relations

The proposed strategy to address Lewis’s concern about material-adequacy involves exploiting the fact that the truth-conditions require there to be an *accessible* world in which the prejacent holds. The definition of the accessibility relation shall thus be altered, and the types of modal operators that are included in the languages of modal predicate logics will be reconstructed.

The notion of an accessibility relation was explained in §1.1, where it was shown how defining a model M requires the stipulation of a binary relation R on worlds. Of course, the stipulated accessibility relation must accord with the modal *wff* true at each $w \in W$; that is, R cannot hold of $\langle w, w' \rangle$ if $w \Vdash_v \Box\phi$ and $w' \Vdash_v \neg\phi$. But beyond that requirement, no guidelines have been issued about the accessibility relations one should stipulate for the consideration of a certain modal claim.

One way to answer Lewis’s objection would therefore be to claim that, in considering the claim $\Diamond(\exists x)(Mx \wedge Bx)$, we generally select an accessibility relation that results in there being no accessible worlds in which $(\exists x)(Mx \wedge Bx)$ holds. That way, the right-hand side of the bi-conditional would correctly be ruled as false, matching the intuitive truth-value of the left-hand side and preserving material-adequacy. However, the problem with such a simple approach is that it has not been explained *why* or *how* the individual considering this modal claim selects this accessibility relation.

In seeking an alternative means to use the accessibility relation to achieve the desired effect, it is first worth considering why it is intuitively false that it is possible for there to be married bachelors. It seems to be the case that this is so because certain *assumptions* about the meanings of the predicates are being made. It is being assumed that the predicates are defined in a way that excludes them from applying to the same objects, and so worlds in which these predicates have different meanings are ignored when considering the claim. The idea of assumptions’ acting to restrict the worlds that are considered to be relevant to the truth-value of a given modal claim will therefore be the key proposal developed in this section.

A set of assumptions A may informally be thought of as the hypotheses endorsed by the individual considering a certain modal claim.⁶⁰ The first condition on the *wffs* within A will be that they are well-formed formulas of the non-modal components of the appropriate language MPL' . Additional conditions will be that A is a *finite, consistent* set of *wffs*. Finally, A may be the empty set if an individual does not assume anything that will constrain the set of accessible worlds.

Instead of proposing that the accessibility relation R should be stipulated in such a way as to guarantee material-adequacy, it will now be proposed that a set of assumptions A is instead selected in a context. Once a set of assumptions has been selected, an assumption-relative accessibility relation R_A will be automatically derived. The definition of a Kripke frame will therefore need to be altered. The following ideas are adapted from Balbiani (1996) and (1999), where parameter-relative accessibility relations are considered for their application to information systems and multimodal logics.⁶¹ For a logic L' and a language MPL' , let a *relative frame* $F_{REL} = \langle W, R \rangle$, where R is a *relative accessibility function* from the sets of finite, consistent *wffs* A of $MPL'_{-\Box, -\Diamond}$ to binary relations on W . Then a *relative model* $M_{REL} = \langle F_{REL}, D, l \rangle$. Within a model, the *wffs* of the assumption-set A will receive interpretations and valuations alongside the *wffs* of the worlds of the model. That is, within a relative model M_{REL} , every interpretation l and variable assignment v will assign the same extension to each variable or predicate α whether α is an element of some world or of some assumption-set.

The relative accessibility function will now be defined. Given a set of *wffs* A as input, a relative accessibility function will yield a *relative accessibility relation* $R(A)$ (abbreviated as R_A), where R_A is a binary relation on W . Given a world Γ as input, a relative accessibility relation will yield the set of all worlds Δ whereby Δ is accessible from Γ relative to A . It shall be stipulated that a world Δ *is accessible from* Γ *relative to* A if and only if Δ is compatible with every *wff* of A . Formally:

$$\langle \Gamma, \Delta \rangle \in R_A \text{ (or } \Gamma R_A \Delta) \iff A \cup \Delta \text{ is } L'\text{-consistent}$$

There cannot be any *wff* of the language $MPL'_{-\Box, -\Diamond}$ that is consistent with a maximal-consistent set of the language MPL' but not already an element

⁶⁰See §7 for more details on the process of selecting assumptions.

⁶¹Though bear in mind a significant difference between my approach and Balbiani's: his assumption-sets are in a language distinct from the modal predicate logic language, whereas mine are not.

of that set. It follows that A can be consistent with Δ if and only if every *wff* of A is already an element of Δ . In short:

$$A \cup \Delta \text{ is } L'\text{-consistent} \iff A \subseteq \Delta$$

If A is the empty set, then every maximal-consistent set for the relevant language MPL' and logic L' will be consistent with A , and hence will be A -accessible. Finally, note that for all A', A'' of the domain of the relative accessibility function, R is a relation such that:

$$R(A' \cup A'') = R(A') \cap R(A''); \text{ and } R(A' \cap A'') = R(A') \cup R(A'')^{62}$$

The preceding discussion should have made the features of relative accessibility relations clearer. However, a problem with the proposed truth-conditions is that the selection of R_A has turned out to be totally arbitrary, since there is no connection between the left side of the bi-conditional and the relative accessibility relation on the right. In brief, it is unclear why the truth of $\Diamond\phi$ at Γ should depend on the presence of A -accessible worlds at which ϕ holds, rather than A' -accessible worlds.

As a solution, let the definition of the language of modal predicate logic from §1.1 be amended with respect to the modal operators:

- MPL' includes a finite number of *relative modal possibility operators* $\Diamond_{A_1} \dots \Diamond_{A_n}$ for each finite, consistent set $A_1 \dots A_n$ of $MPL'_{-\Box, -\Diamond} \subset MPL'$.

The dual of the relative possibility operator will be the relative necessity operator, defined in the usual way. A claim ϕ prefixed by a modal operator O_A should be read as ' ϕ is possible (/necessary) relative to A '. It is being proposed that, when a modal claim is issued at a world, the modal term is tacitly relativised to an assumption-set. The addition of such relative modal operators may be seen to give us the following truth-conditions:

$$M_{REL}, \Gamma \Vdash_v \Diamond_A \phi \iff \exists \Delta \in W : \Gamma R_A \Delta \text{ and } M_{REL}, \Delta \Vdash_v \phi$$

Now, R_A is selected as the relevant accessibility relation, rather than $R_{A'}$, due to the fact that the modal operator prefixing the claim to be assessed has the appropriate subscript.

⁶²Balbani (1996) p.30.

It should be noted that this definition of relative accessibility relations entails that, if $\Gamma R_A \Delta$ for *some* Γ , then $\Gamma R_A \Delta$ for *any* Γ ; for the only condition on $\langle \Gamma, \Delta \rangle$ standing in the R_A relation is that $A \subseteq \Delta$. At first, this might seem to be an unwelcome result, since flexible properties of the accessibility relation may no longer be stipulated. However, it should be recalled that one of my aims is to model the process by which modal claims are issued. Therefore, if arguments pertaining to the suitability of a less flexible accessibility relation for modelling modal discourse are compelling, then the inflexibility of relative accessibility relations should not be perceived to pose a problem.

First consider the matter of reflexive relations. There will be some cases where $A \cup \Gamma$ is consistent, and other cases where $A \cup \Gamma$ is inconsistent. Therefore R_A will be reflexive at some worlds, and non-reflexive at others. The situation is similar for symmetry. I would argue that this reflects modal discourse: in general, individuals attempt to make true assumptions, and will subsequently invoke a reflexive accessibility relation if they succeed and inadvertently invoke a non-reflexive relation if they fail; in some cases, such as when issuing counterfactual modal claims, it seems to me that individuals deliberately try to employ false assumptions, and they therefore invoke non-reflexive accessibility relations if they succeed. It should also be borne in mind that *some* reflexive relative accessibility relation can always be identified; it is simply the case that one might need to employ a relation that is relative to a different assumption-set A' , or consider a world that differs from Γ only insofar as $A \cup \Gamma$ is consistent.

Transitivity is a more complex matter. It is clear that a formula $\Box_A \Box_A \phi$ will hold at a world Γ if and only if $\Box_A \phi$ does, since the A -accessible worlds from those that are A -accessible from Γ will be the same set as those worlds that are A -accessible from Γ . No truly non-transitive relative accessibility relation can be identified according to the current approach, then. However, despite the fact that non-transitivity is a useful property to reflect in the semantics of modal logics, it is no great loss to constrain it if a speaker would never issue a claim of the form $\Box_A \Box_A \phi$ instead of $\Box_A \phi$. I would suggest that all modal claims that seemingly employ a non-transitive accessibility relation in fact involve ‘pseudo-non-transitivity’, in the sense that the modal operators are associated with distinct assumption-sets.⁶³ For note that a

⁶³For example, consider the sort of exchange that might support the claim that non-transitive accessibility relations are required to analyse modal discourse:

Red: ‘It might be possible to have your sentence reduced’;

Prim: ‘So you’re saying it’s possible for my sentence to be reduced?’;

Red: ‘No, it would be premature to say that ... I’ll need to consult the case law’.

formula $\Box_A \Box_{A'} \phi$ may hold at Γ when either $\Box_A \phi$ or $\Box_{A'} \phi$ *fails* to hold. The first scenario arises in cases where ϕ holds in every A' -accessible world but not every A -accessible one; and the second scenario arises when $A \cup \Gamma$ is inconsistent but one of the assumptions in A' involves an expression that is indexical on the world Δ that A' is at,⁶⁴ with the truth of $\Box_{A'} \phi$ depending on Δ 's being consistent with A . Psuedo-non-transitive accessibility relations between worlds *can* always be captured, if one selects assumption-sets and worlds carefully. In summary, I do not perceive it to be problematic that relative accessibility relations are more constrained than the accessibility relations within Kripke semantics, since they are arguably sufficient to accommodate the modal claims actually expressed by speakers.

In this subsection, assumption-relative accessibility relations and assumption-relative modal operators were defined and used to modify the definitions of §1.1. The account that is being endorsed has been altered: non-relative modal operators have been eliminated from the languages of modal predicate logic, and replaced with assumption-relative modal operators. The way in which these alterations help guarantee material adequacy will be explained in the next subsection.

0.4.4 A Materially-Adequate Definition of ‘Possible World’

In order to see how the notion of assumption-relative accessibility relations helps preserve material-adequacy in the face of the issue identified by Lewis, the earlier example concerning the possibility of married bachelors shall be returned to. The truth-conditions for this problematic claim were:

$$M, \Gamma \Vdash_v \Diamond(\exists x)(Mx \wedge Bx) \iff \exists \Delta \in W : \Gamma R \Delta \text{ and } M, \Delta \Vdash_v (\exists x)(Mx \wedge Bx)$$

It was conceded that, on the account previously endorsed, material-adequacy was lost due to the existence of a maximal-consistent set that has $(\exists x)(Mx \wedge Bx)$ as an element. In the previous section, accessibility relations between sets were looked at in more detail, and assumption-relative accessibility relations were defined. The new, assumption-relative truth-conditions for the

I would read this exchange as involving two different flavours of modality, such that a suitable paraphrase is: ‘It is compatible with my knowledge that the world is such that the law is compatible with your sentence’s being reduced’. Hence this is an instance of pseudo-non-transitivity.

⁶⁴See §6.2 and §7 for clarification regarding what it means for an assumption-set to be *at* a world.

problematic claim will be the following:

$$M_{REL}, \Gamma \Vdash_v \Diamond_A(\exists x)(Mx \wedge Bx) \iff \exists \Delta \in W : \Gamma R_A \Delta \text{ and } M_{REL}, \Delta \Vdash_v (\exists x)(Mx \wedge Bx)$$

The relevant set of assumptions will be fixed by the individual who is associated with the claim.⁶⁵ He might have A as the empty set, in which case there will be an accessible world in which the problematic embedded claim holds. However, he may alternatively make assumptions about the meanings of the predicates, in order to restrict the relative modal operator to the worlds in which the predicates M and B never apply to the same domain-member:

$$A = \{(\forall x)(Bx \rightarrow \neg Mx)\}^{66}$$

The assumption set A contains a single *wff* that bars any individual from having M and B simultaneously predicated of it. This is a *meaning postulate*, which is an axiom that imposes restrictions on the interactions of the predicates of a language. Since the *wff* included in A is inconsistent with any set that includes the formula $(\exists x)(Mx \wedge Bx)$, it can be guaranteed that there will be no accessible world in which this formula holds; so the right-hand of the bi-conditional will be false, in accordance with the falsity of the left-hand side given such assumptions about the meanings of the predicates.

However, it has only been shown that some assumptions A may be employed that rule out worlds that make the formula $\Diamond_A(\exists x)(Mx \wedge Bx)$ true. To avoid Lewis's objection, it must be shown that there is a way to *guarantee* that bi-conditionals featuring this claim will be materially-adequate.

There are two ways to use the apparatus of assumption-relative accessibility relations to argue that material-adequacy for the claim 'It is possible for there to be married bachelors' is guaranteed in the face of Lewis's objection. The first will involve holding that there are some circumstances in which an assumption-set A is selected that renders $\Diamond_A(\exists x)(Mx \wedge Bx)$ true, since it is possible in the broadest sense of the word for there to be married bachelors. The advocate of this first strategy will say that the reason claims about the possibility of married bachelors intuitively seem false is that the situations in which we wish to consider the broadest sense of

⁶⁵ At the moment, I am being deliberately vague about exactly which individual this is. See §7 for further clarification.

⁶⁶ Carnap (1988) p.223.

possibility and the entire space of worlds are incredibly rare. Normally, the set of assumptions is non-empty, and at least includes meaning postulates about the relevant terms.

The second strategy will involve agreeing with Lewis that $\Diamond_A(\exists x)(Mx \wedge Bx)$ is always false. The advocate of this second approach obviously needs to reconcile this idea with the fact that an unbounded accessibility relation will yield maximal-consistent sets in which the embedded claim is represented as true. To do this, he will hold that the entire space of possible worlds is not given by all maximal-consistent sets for a specific logic, language and domain; instead, it is given by all maximal-consistent sets for a logic, language and domain *that are compatible with assumption-sets featuring certain meaning postulates of the relevant language*. I doubt the set of appropriate meaning postulates for a language is sufficiently clearly defined to allow the desired assumption-sets to be generated by the inclusion criteria of *being an appropriate meaning postulate*. More plausibly, an advocate of this second strategy would have to either individually list the meaning postulates that he takes to yield the absolute space of worlds, or find some ingenious means of specifying the general form that the desired meaning postulates will take. This strikes me as a significant challenge; still, if successful, his approach would mean that the set of assumptions A may never be empty.

It should be clear that both strategies converge with regards to endorsing the technique of assumption-relative accessibility relations for narrowing down the space of accessible worlds. The difference arises in how narrow they choose to make the absolute space of worlds. With regards to the choice between these two strategies, my preference would be for the first strategy. This is the case because I personally find it more undesirable to hold that modal logic incorrectly characterises many impossible worlds as possible than that it is possible (in an extremely broad, generally useless way) for there to be married bachelors. Moreover, I think the practical challenge noted for the second strategy causes more difficulties than it solves. However, either strategy is compatible with the current account. Provided one of the strategies seems tolerable, we may put aside the issue of where the boundaries of the absolute space of worlds fall and leave it as a matter of preference for individual theorists.

It will now, at last, be argued that assumption-relative accessibility relations guarantee material-adequacy for *every* modal predicate logic claim. If an individual believes that a certain modal claim is true or false, but this intuitive truth-value does not appear to be reflected on the right-hand side of the appropriate bi-conditional, then this mismatch between the truth-values may either be attributed to certain assumptions' being tacitly employed in

determining the truth-value of the modal claim that are not reflected on the right-hand side; or attributed to the individual's making a mistake about what is compatible with certain assumptions. In the first scenario, the individual considering the claim may be analysed as using a *MPL'* translation that features a modal operator O_{A_1} , which they mistakenly believe to be correlated with a claim about the A_2 -accessible maximal consistent sets, where $A_1 \neq A_2$. This should eliminate any disparity between the truth-values on the right and left sides: for provided the individual considering the modal claim is aware of the assumptions they are employing, in addition to being correct about the possibility or necessity of the embedded claim relative to the assumptions, then the truth-value they assign to the modal claim on the left side of the bi-conditional will match the truth-value of the corresponding right side. In §4.2 it was concluded that the truth-value of the left-hand side of a particular instance of the bi-conditional cannot be determined subsequently to the truth-value we assign to the right-hand side (the 'non-arbitrariness' constraint), and this requirement has been upheld on the current account. For the assumptions of the individual associated with the modal claim are now prioritised, rather than the worlds that happen to be in a given set.

In this subsection, the material-adequacy of the account that has been developed was argued for. This challenges Lewis's claim that linguistic ersatz accounts that make use of a syntactic definition of 'consistency' generate a chaotic space of worlds. The cost of preserving material-adequacy in this way is commitment to the idea that something is possible or necessary only relative to certain assumptions. Rather than merely being a technical convenience, this appears to accord well with our everyday use of modal claims.⁶⁷

To conclude: an account has been proposed in this section that has been argued to meet the material-adequacy and non-circularity conditions. The results of §4, if convincing, therefore show that the developed account is a successful reductive account, according to the definition of §2.1. The motivation to adopt a successful reductive, linguistic ersatz account, if one could be found, was argued for in §3.2. It has just been seen that one can be found. It follows that we should adopt it.

⁶⁷See Kratzer (1991). §7 also lends support to this assertion.

0.5 Extending the Account to Metaphysical Modal Claims

An idea that might derive from the assumption-relativity of accessibility relations is that the *flavour* of modality associated with a modal claim is related to the type of assumptions employed. For example, the claim that it is not possible for there to be married bachelors is generally taken to be an *analytic* necessity. A necessary truth is typically defined as an instance of analytic necessity if it holds only by virtue of the meanings of the non-logical terms involved in the claim. In giving the correct analysis of the modal claim affirming the possibility of married bachelors in §4.4, only a meaning postulate about the relevant predicates had to be assumed. We may therefore identify certain modal claim as analytic on the basis of the contents of the relevant assumption-sets. Similarly, we might expect that *epistemic* modal claims would be issued relative to assumption-sets that include what some individual takes herself to *know*, *deontic* modal claims would be made relative to assumption-sets that include *rules* identified by some individual, and so on. It therefore seems that metaphysical modal claims could also be characterised on the basis of the contents of the assumption-sets that constrain their interpretation. This shall be the topic of the current section.

I shall begin by attempting to handle metaphysical modal claims without modifying the current account (in §5.1), and then by implementing the minimal modification of introducing constants into the modal predicate language (in §5.2). Yet in §5.3, it will be found that the only way to accommodate metaphysical modal claims is to implement the more significant modification of postulating that an essentialist predicate appears in the assumption-sets for such claims. This predicament will then be analysed further in the following section.

0.5.1 Attempt 1: The Current Account Unmodified

Let us take the metaphysical modal claim ‘It is possible for Adam to have been a snake’ and see how far the account proposed so far gets us. Given a suitably related claim in a language of modal predicate logic,⁶⁸ we have the

⁶⁸Obviously, the claims ‘It is possible for Adam to have been a snake’ and ‘There is a man for whom it is possible that he is a snake’ are not equivalent, since the latter but not the former is true if Noah is possibly a snake. However, at this point there is no clear way to express predications involving named individuals. Since the former claim entails the latter, any assumptions that rule out worlds in which the latter holds also suffice to rule out the former.

following truth-conditions:

$$M_{REL}, \Gamma \Vdash_v (\exists x)(Man(x) \wedge \Diamond_{A'} Snake(x))$$

$$\iff \text{for some } x\text{-variant } v' \text{ of } v \text{ at } \Gamma, \Gamma \Vdash_{v'} Man(x) \text{ and } \Gamma \Vdash_{v'} \Diamond_{A'} Snake(x)$$

$$\iff \text{there is some } \Delta \in W : \Gamma R_{A'} \Delta \text{ and } M_{REL}, \Delta \Vdash_{v'} Snake(x)$$

The pressing question, then, is what assumptions we should include in A' if we want to rule out the metaphysical possibility of a man's being a snake. It shall be seen that, within the current system, there are neither non-modal nor modal expressions that achieve the desired effect.

First consider the assumptions that could be utilised if we uphold the requirement to exclude modal operators from the assumption set. We might try to rule out worlds in which a man is a snake as follows:

$$1. A' = \{(\forall x)(Man(x) \rightarrow \neg Snake(x))\}$$

Despite this being a reasonable meaning postulate regarding the extensions of the two predicates, this clearly does not help with the current case: for our concern is not to rule out worlds in which an individual is simultaneously a man and a snake. Rather, we wish to impose restrictions on the properties individuals may have *across* worlds, and it seems that no non-modal expression currently available to the account will enable this.

Suppose we instead attempt the following:

$$2. A' = \{(\forall x)(Man(x) \rightarrow \neg \Diamond_{?} Snake(x))\}$$

There are two ways of interpreting the modal operator $\Diamond_{?}$, and either way causes trouble for the current account. Firstly, we might take $\Diamond_{?}$ to be an absolute possibility operator \Diamond . But then it turns out that we have a notion of modal operators that fail to be relative to non-empty assumption-sets, and this notion is distinct from broad logical consistency (since $Snake(x)$ clearly does not express a logical contradiction). This is incompatible with the conclusions I have been arguing for, and is therefore tantamount to abandoning the current search for a reductive account of metaphysical modal claims. If, alternatively, we take $\Diamond_{?}$ to be a relative modal operator, then we must determine the assumption set to which it is relative. It is clear that, if we import allusions to other assumption sets A'' into the assumptions set A' , then a regress can be obtained: that is, there would be nothing to rule out

cases where individuals employ an infinite series of assumption-sets, which never terminates with the assumption-set free of modal operators required to ground the whole process. If we were to allow modal claims to occur in A' that are relative to A' itself, then the question once more emerges of what non-modal assumptions we can put into A' to rule out worlds in which $\Diamond_{A'} Snake(x)$ holds. The first problem with 2. is therefore that construing $\Diamond_?$ as either an absolute or relative modal operator spells trouble for the overarching account.

There is a second problem with 2. (in case the first problem does not seem serious enough): a puzzling sense of triviality arises if modal claims are allowed to appear in assumption sets. We are assigning a truth-value to $\Diamond_{A'} Snake(x)$ on the basis of the presence or absence of an assumption that $\Diamond_? Snake(x)$. We thereby seem to be assuming the truth or falsity of a modal claim in order to determine the truth-value of a related modal claim. This contrasts with the earlier described case, where the truth-value of a modal claim is determined relative to pre-existing *non-modal* assumptions.

In light of these issues with 1. and 2., the prospect of extending the developed account to handle metaphysical modal claims does not look promising. However, in the following subsections I wish to explore other means of salvaging the project.

0.5.2 Attempt 2: Introducing Constants into the Account

I take the previous subsection to convincingly demonstrate the reasons that the restriction against modal claims in the assumption-set must remain. Yet, as we have seen, this restriction means that *de re* modal claims, such as $(\exists x)(Man(x) \wedge \neg \Diamond_{A'} Snake(x))$, cannot be treated identically to *de dicto* ones like $\neg \Diamond_{A'} (\exists x)(Married(x) \wedge Bachelor(x))$. Roughly, let us say that ‘A sentence containing a modal operator counts as *de re* if and only if it involves either quantification into a modal context or the presence of a rigid designator within the modal context’.⁶⁹ Since the modal operator has scope over the entire formula in the case of *de dicto* claims, we may delete the modal operator and rule out worlds in which the embedded non-modal claim holds through appropriate non-modal assumptions. Yet for *de re* claims in which quantifiers out-scope modal operators, we cannot remove the modal operator without affecting the structure of the modal claim. Moreover, preserving the modal operator within the assumption-set is barred for the reasons previously noted.

⁶⁹Mackie (2006) p.7.

One might conclude that the language of modal predicate logic should be extended in order to allow *de re* claims to be formulated by means of *rigid constants*. Let the syntax and semantics of modal predicate logics therefore be extended as follows:

1. To the set of terms, add a finite set of *constants* $\{c_1 \dots c_n\}$.
2. To the set of atomic *wffs*, add all expressions $P(c_1 \dots c_n)$ and $c_1 = c_n$, where P is an n -place predicate and $c_1 \dots c_n$ are individual constants.
3. The interpretation function l is required to assign to each constant c some $a \in D(F)$, relative to each $\Gamma \in W$.
4. $l(c, \Gamma) = l(c, \Delta)$ for all $\Gamma, \Delta \in W$.
5. Define the general interpretation function $(v * l)$, such that for a variable x , constant c and world Γ :
 - (a) $(v * l)(x, \Gamma) = v(x)$
 - (b) $(v * l)(c, \Gamma) = l(c, \Gamma)$
6. Where α is a variable or constant, alter the clauses of the truth-conditions pertaining to atomic formulas as follows:
 - (a) $M_{REL}, \Gamma \Vdash_{v * l} P(\alpha_1 \dots \alpha_n) \iff \langle (v * l)(\alpha_1, \Gamma) \dots (v * l)(\alpha_n, \Gamma) \rangle \in P$
 - (b) $M_{REL}, \Gamma \Vdash_{v * l} \alpha_1 = \alpha_n \iff (v * l)(\alpha_1, \Gamma) = (v * l)(\alpha_n, \Gamma)$

The general interpretation function is introduced to reflect the fact that terms are interpreted differently depending on whether they are variables or constants, with the truth-conditions for atomic formulas then changed accordingly. The fourth condition stipulates that all constants are *rigid*, in the sense that they designate the same individual at every world. Constants should therefore only be used as translations of *proper names* within this system, with definite descriptions' being expressed through the use of bound variables and predicates. It should also be noted that, although 'Adam' will denote the same individual at every world, the individual it denotes at some world may lack all of the properties associated with the actual referent of 'Adam'. Assumptions will be required to restrict the set of accessible worlds to those in which the referent of 'Adam' possesses particular qualities. Finally, note that nothing stipulated so far requires it to be the case that $l(c, \Gamma) \in D(\Gamma)$.

Let us now see how the extended language helps the search for suitable assumptions for metaphysical modal claims. Suppose we try to rule out worlds in which Adam is a snake as follows:

$$3. A' = \{(Man(adam)); (\forall x)(Man(x) \rightarrow \neg Snake(x))\}$$

At first glance, this seems promising: for it looks as if A' will not be compatible with any Δ such that $\neg Snake(adam) \in D(\Delta)$. However, this is not so, due to the presence of worlds in which Adam is not a domain-member. For note that, when $l(adam) \notin D(\Delta)$, the quantifiers within the meaning postulate have no effect on the referent of $adam$, so A' allows it to be the case that $(Man(adam) \wedge Snake(adam)) \in \Delta$; yet this was precisely the formula that we were trying to exclude from the set of accessible worlds when making such assumptions.

It should be recalled that, in §1.1, *varying* and *constant* domain frames were defined. By considering the possibility of its being the case that $I(c, \Delta) \notin D(\Delta)$, it has been tacitly assumed that the current account is compatible with varying domain frames. I will now renounce the tacit tolerance of different domain types, and endorse a *constant domain framework* for the current account of modal claims. It initially seems counter-intuitive to hold that everything necessarily exists; however, a predicate ‘*Concrete*’ may be defined, such that a domain-member is concrete just in case it exists in an everyday sense of the word. The concrete individuals of the actual world will be abstract in other scenarios (i.e. those situations in which we would be tempted to say that they do not exist), and perhaps there are a host of actually non-concrete objects. It is preferable for technical reasons to treat non-concrete objects exactly the same as concrete ones, in the sense that they will be either P or *not- P* (and not both simultaneously) for every predicate P .⁷⁰ Admittedly, my motives for endorsing this constant domain

⁷⁰This strikes me as the best of three options with regards to predicating properties of non-concrete individuals. The second option would be to hold that, at worlds at which Adam is not concrete, then $P(adam) \notin \Gamma$ and $\neg P(adam) \notin \Gamma$, for any predicate P (except, presumably, for the predicate $\neg Concrete$). The problem with this view is that it violates maximality, meaning that the construction process for possible worlds needs to be rethought. Construing possible worlds as non-maximal consistent sets would result in the absolute space of worlds being defined far less neatly. The third option would be to hold that, at worlds at which Adam is not concrete, then $P(adam) \notin \Gamma$ but $\neg P(adam) \in \Gamma$, for any predicate P . Whilst this view initially seems to be the most intuitively compelling, it is technically problematic. One issue is that there is no way to syntactically identify such denials of properties. It would not suffice to allow only those *uff* in which each predicate affirmed of Adam is prefixed by a \neg symbol to occur in Γ , since this would

perspective are mainly theoretical. However, there are also good philosophical reasons for adopting a constant domain semantics.⁷¹ All of this means that $l(adam) \in D(\Gamma)$ for all $\Gamma \in W$, and A' rules out worlds in which Adam is both a man and a snake in the intended manner.

Yet 3. still does not suffice for current purposes. The problem is that my account is intended to model the process by which speakers assume certain facts when issuing modal claims. Yet it is fairly implausible to suppose that a speaker renders the claim ‘It is impossible for Adam to have been a snake’ true by assuming that ‘Adam is a man’. Some premises appear to be missing, and such premises should be explicitly included in the assumption-set rather than being implicitly employed. Furthermore, there are other claims about the actual properties of Adam that a speaker might assent to the truth of, such as ‘Adam is bearded’. Yet the speaker need not automatically treat the metaphysical possibility of Adam’s lacking each property equally. A way of differentiating between the properties a speaker believes actually hold of an individual, and those she believes actually hold in a modally significant way, should therefore be sought. Nevertheless, the introduction of constants represents progress, since it allows *de re* modal claims to be targeted by the assumption set.

0.5.3 Attempt 3: Introducing an Essentialist Predicate into the Account

Suppose we define an *essentialist predicate* \mathbb{E} , which may prefix an atomic formula featuring a constant, in order to rule out worlds in which the negation of that atomic formula holds. The only formal information about its interpretation will be the following conditional:

$$\mathbb{E}(P(\alpha_1 \dots \alpha_n)) \rightarrow P(\alpha_1 \dots \alpha_n)$$

where P is an n -place predicate and $\alpha_1 \dots \alpha_n$ are constants. This tells us, for

permit $\neg\neg P(adam)$ to occur in Γ , which entails that $P(adam) \in \Gamma$. A solution might be to identify denials of properties semantically, holding that $l(adam) \notin l(P, \Gamma)$ for any predicate P . However, we then get the result that $l(adam) \notin l(Unmarried, \Gamma)$. A non-concrete individual will therefore fail to be in the extension of predicates that already express the absence of a property, resulting in the truth of counter-intuitive sentences such as $(\neg Married(adam) \wedge \neg Unmarried(adam))$ (the truth of which surely negates the fact that the third approach initially seems to be the most intuitive). In light of these considerations, I would argue that the first option is preferable to the others, since it is both technically simpler and no less counter-intuitive than them.

⁷¹For example, see Williamson (2013).

example, that if it is claimed that Adam is essentially a man, then it is being assumed that Adam is a man. The other direction clearly does not hold, since the point that emerged from the previous subsection is that affirming that an individual possesses a particular property is insufficient for this property to automatically affect the metaphysical claims issued about that individual. This essentialist predicate could then be used in assumption sets like so:

$$4. A' = \{\mathbb{E}(Man(adam)); (\forall x)(Man(x) \rightarrow \neg Snake(x))\}$$

The modal claim $M_{REL}, \Gamma \Vdash_v (\exists x)(Man(x) \wedge \Diamond_{A'} Snake(x))$, or the variant $M_{REL}, \Gamma \Vdash_v \Diamond_{A'} Snake(adam)$, will then come out false relative to the assumption-set A' ; for the definition of \mathbb{E} ensures that $Man(adam)$ holds at all A' -accessible worlds, and the constant domain semantics means that the second formula of A' excludes worlds at which $Snake(adam)$ is true.

One pressing question is whether the predicate will itself be a symbol of the modal predicate language MPL' . Initially, consider several reasons to answer this question negatively: MPL' is supposed to be first-order, and introducing a second-order predicate would result in the typical problems with completeness. Moreover, giving ‘truth-conditions’ for this predicate merely involves the provision of a *conditional*, rather than the usual *bi-conditional*. Finally, adding it to the language MPL' would mean that, due to maximality, each world would have to either include $\mathbb{E}(P(\alpha_1 \dots \alpha_n))$ or $\neg \mathbb{E}(P(\alpha_1 \dots \alpha_n))$ as an element for every atomic featuring constants alone. One consequence of this is that, for each atomic $P(\alpha_1 \dots \alpha_n)$, we would have two copies of each world that differ only with regards to the presence of either the essentialisation of this claim or its negation. Some individuals might find it counter-intuitive that worlds could be identical with regards to all facts other than whether or not some property is essential to an individual. I find this tolerable, but what I find more difficult to tolerate is that the set representing *the actual world* would then include either $\mathbb{E}(P(\alpha_1 \dots \alpha_n))$ or its negation for each essentialist claim.⁷² I have some doubts with regards to the robust thesis that there are objective facts about the properties that are and are not actually essential to an individual. These considerations appear to motivate the exclusion of the essentialist predicate from the language MPL' . According to this view, the essentialist predicate that may

⁷²I am here presupposing a picture whereby there is a maximal-consistent set that accurately represents the actual world, though only an omniscient creature could determine which set this is.

appear in assumption-sets is ‘special’ insofar as individuals are presupposing a fact that is not entirely expressible in their language. Given that metaphysical modal claims are often perceived to be ‘special’ amongst modal claims, this seems appropriate. Another consequence of this view is that an assumption-set may now be either consistent or inconsistent with a given world *without being a subset of that world*. For if the essentialist predicate is barred from being an element of worlds, then no assumption-set featuring it would be a subset of any world. However, this does not appear to be a significant modification, since consistency and inconsistency continue to achieve the desired effect. An assumption-set that includes $\mathbb{E}(P(\alpha_1 \dots \alpha_n))$ will be consistent with any world that excludes $\neg(P(\alpha_1 \dots \alpha_n))$ (and hence, by maximality, includes $(P(\alpha_1 \dots \alpha_n))$).

However, suppose that one does not find these considerations particularly compelling. Suppose one thinks that it is acceptable for the language MPL' to have a second-order fragment, and that truth-conditions in the form of a conditional are unusual but not unacceptable; furthermore, suppose one finds the idea that there are objective essentialist facts that hold at worlds to be intuitively compelling. In this case, one may include the essentialist predicate in the language MPL' , and continue to uphold the idea that assumption-sets are subsets of the worlds with which they are consistent, without causing my account any real trouble.

The proposal, then, is that individuals make assumptions about the essential properties of individuals when considering metaphysical modal claims. Suggesting that speakers merely assume that Adam *is* a man was held to be implausible. The current suggestion is that speakers assume that Adam *is essentially* a man, and rule out worlds in which he fails to be a man on this basis. Yet there is a significant objection to this proposal: one might suspect detailed analysis to reveal \mathbb{E} to be reducible to *modal* terms. In such a scenario, covert modal operators would be appearing in assumption sets, and the issue of whether such operators are absolute or relative would merely be obscured by their tacit nature. As explained earlier, the appearance of modal operators in assumption sets (whether absolute or relative) proves catastrophic for the current reductive approach. It therefore must be demonstrated that \mathbb{E} is not reducible to modal terms. This shall be the topic of the following section.

0.6 Accepting \mathbb{E} as a Primitive

Having found that we need to endorse an essentialist predicate in the previous section, the nature of this predicate shall now be determined. In §6.1, the traditional and Finean perspectives on essentialist predicates will be discussed. In §6.2, it shall be argued that, as Fine claims, modal terms cannot be employed to give a reductive definition of this essentialist predicate. However, in §6.3, I will argue, contra Fine, that an analysis of the essential predicate in terms of modal expressions will at least be materially-adequate. The arguments from §6.2 and §6.3 must go through if the essentialist predicate is to allow my account to be extended to metaphysical modal claims. Finally, in §6.4, I consider the unwelcome consequences of taking the essentialist predicate as a primitive, and assess the implications for my account.

0.6.1 Essential Properties: Traditional and Finean Accounts

I shall begin by providing some background on essentialism within the literature. There are several perspectives traditionally adopted towards essential properties. *Aristotelian essentialism* is the view that ‘some of the attributes of a thing . . . may be essential to the thing, and others accidental’.⁷³ We may further distinguish *non-individuating* from *individuating* Aristotelian essentialism: the former is the view that the only attributes essential to a particular thing are those that are essential to everything; the latter holds that some object possesses a set of essential attributes that differs from those of some other object in the domain, allowing us to individuate the objects on the basis of their divergent essential properties (though it is left open whether some objects possess unique individual essences, or whether essential properties will only ever establish equivalence classes of domain-members).⁷⁴ Finally, *haecceitism* denies that any attributes of any object are essential to it. An unanalysable essential identity (often construed as a reflexive identity relation) that does not supervene on properties is instead postulated.⁷⁵

Normally, in discussions on the topic of essentialism, metaphysicians will endorse a particular one of these differing view-points as correct, before developing an account of which properties are essential to which objects (assuming they endorse a view other than haecceitism). This contrasts with the current discussion of essentialism. For I do not wish to suggest which

⁷³Quine (1966) p.173.

⁷⁴Parsons (1967)

⁷⁵Rosenkrantz (1993) p.1.

expressions (if any) the essentialist predicate correctly applies to. My aim is to model the process by which metaphysical modal claims are issued and interpreted. My account therefore accommodates the sorts of claims made by both essentialists and haecceitists, allowing *both* sorts of claims to be true relative to certain assumption-sets.⁷⁶ That is, if an individual assumes certain essentialist claims, then only those worlds in which the relevant properties hold of the objects concerned will be accessible; and if a haecceitist refrains from assuming any essentialist claims, then worlds in which diverse properties are attributed to each object will be accessible.

Separate from the question of the essential properties (if any) possessed by an individual is the question of what it means for a property to be *essential*. Traditionally in contemporary analytic philosophy, ‘essential property’ has been defined modally.⁷⁷ A typical modal definition would be this:

Traditional: P is an essential property of $a \iff a$ is necessarily P .⁷⁸

Such a definition is *modal* by virtue of including the modal expression *necessarily* on the right-hand side. Recalling the definition of reduction in §2.1, this constitutes a reductive definition of ‘essential property’ provided both the material-adequacy and non-circularity conditions hold.

Recently, however, this modal definition has been challenged. Kit Fine presents two main arguments against it.⁷⁹ Firstly, *necessary truths that are entailed by the existence of a particular object will come out as essential properties of that object, even if they are intuitively unconnected to the nature of that object*. For example, Socrates necessarily belongs to the set whose sole

⁷⁶Whether there is a particular privileged assumption-set for the analysis of metaphysical modal claims is a question I tackle in §7.

⁷⁷Fine (1994a), p.3. For example, such a notion of essential properties is assumed in Marcus (1967) and Kripke (1980).

⁷⁸Fine also discusses an *existentially conditioned variant*, ‘ P is an essential property of $a \iff a$ could not exist without being P ’. The reason I have restricted my attention to the categorical version is that it makes more sense within the context of the current framework. In short, if we were to endorse the existentially conditioned variant and interpret ‘exist’ to mean ‘is a domain-member’, then the necessary existence of all entities within this constant domain system would cause the variant to collapse into the categorical version. On the other hand, if ‘exist’ were to be interpreted as ‘is concrete’, then the existence conditionalisation is not required due to the fact that the current account allows a to be P whether or not a is concrete. Assuming that a is essentially P would result in the only accessible worlds’ being those in which a is P whether or not a is concrete; and if one wished to focus only on the worlds in which a is also concrete, a further assumption could be introduced.

⁷⁹Fine (1994a), pp.4-6.

member is Socrates if he exists (according to standard views within set theory), so assumed material-adequacy renders ‘*Belonging to singleton Socrates if Socrates exists*’ an essential property of Socrates.⁸⁰ However, ‘Strange as the literature on personal identity may be, it has never been suggested that in order to understand the nature of a person one must know to which sets he belongs.’⁸¹ Secondly, *every necessary truth will turn out to be an essential property of any object, even if that object is irrelevant to its obtaining*. That is, in every world in which Socrates exists (in either sense of the word), nothing is both blue and not-blue. The assumed material-adequacy once more entails the truth of the left-hand side with the appropriate substitution. Yet Socrates’ essence seemingly fails to include anything about blue and not-blue objects, or about non-contradiction in general.

Fine takes these criticisms of the traditional definition of ‘essential property’ to motivate the following alternative definition:

Finean: P is an essential property of $a \iff$ it helps answer the question: ‘What is a ?’.⁸²

There are several features that should be noted about this perspective. Firstly, it suggests that the notion of essence ‘is not to be understood in modal terms or even to be regarded as extensionally equivalent to a modal notion.’⁸³ In other words, the reductive definition traditionally attempted fails. Fine’s two putative counter-examples indicate that he takes it to fail due to a violation of the *material-adequacy* condition. That is, the right-hand side may sometimes be true when the corresponding left-hand side is false. At the same time, Fine accepts that the left-to-right direction of the traditional definition holds: that is if P is an essential property of a , then a could not exist without being P .⁸⁴ Furthermore, Fine’s perspective ‘sees real definition rather than *de re* modality as central to our understanding of the concept’.⁸⁵ By ‘real definition’, Fine is referring to our ability to ‘de-

⁸⁰Fine’s initial formulation of this objection applies to the existentially conditioned variant, and I have followed his recommendations of how to adapt it to apply to the categorical version of the traditional view (Fine (1994a) p.6). Of course, there is the ambiguity in the word ‘exists’ once more. If it means ‘is a domain-member’ then this addition is trivial.

⁸¹Fine (1994a) p.5.

⁸²Roca-Royes (2011) p.66.

⁸³Fine (1994a) p.2.

⁸⁴Roca-Royes (2011) p.66.

⁸⁵Fine (1994a) p.3.

fine an object, or say what it is'.⁸⁶ He does not go into much detail about how we define an object (e.g. it is not clear whether an essential property belongs to an object by virtue of an individual's definition of that object, due to a broad consensus regarding the definition of that object, or because of objective features that are 'defined' by being singled out as of importance); however, he notes that this definition of 'essential property' 'may not provide us with an analysis of the concept, but it does provide us with a good model of how the concept works'.⁸⁷ Finally, he holds that 'far from viewing essence as a special case of metaphysical necessity, we should view metaphysical necessity as a special case of essence ... The metaphysically necessary truths can then be identified with the propositions which are true in virtue of the nature of all objects whatever'.⁸⁸

The key facts that Fine endorses, then, are these:

- i) it *is* the case that, if P is an essential property of a , then a is necessarily P ;
- ii) it *is not* the case that, if a is necessarily P , then P is an essential property of a ;
- iii) P is an essential property of a *if and only if* it helps answer the question: 'What is a ?';
- iv) there is no reductive definition of 'essential property';
- v) metaphysical necessity may be reductively defined through making use of the term 'essential property'.

0.6.2 Defending the Irreducibility of \mathbb{E} to Modal Terms.

As earlier explained, I need to ensure that the essentialist predicate \mathbb{E} cannot be reduced to modal terms, in order to avoid compromising the reductive aspects of my account. Endorsing Fine's account of essence would therefore be extremely useful. My positions on the key facts of his account may be summarised thus:

- I *endorse* i);

⁸⁶Fine (1994a) p.2.

⁸⁷*Ibid.* p.3.

⁸⁸*Ibid.* p.9.

- I *deny* ii);
- I *endorse* iii), although it has not been ruled out that different individuals may answer the question ‘What is a ?’ in different ways, causing the essential properties of an object to be relativized to individuals’ assumptions;
- I *endorse* iv), though this will cause an interesting problem later;
- I *endorse* v), in the sense that metaphysical necessity may be reductively defined *partially* through making use of the term ‘essential property’ (in addition to other aspects of the system, such as maximal-consistent sets and assumption-sets).⁸⁹

I shall start by discussing my responses to i) and ii). The traditional definition of ‘essential property’ may be reformulated as follows:

Traditional Reformulated: $\mathbb{E}(P(\alpha)) \in A' \text{ at } \Gamma \iff M_{REL}, \Gamma \Vdash_{v*l} \Box_{A'} P(\alpha)$

So far, I have not indicated what it means for an assumption-set A' to be *at* Γ . I will later suggest that assumption-sets are generated by the application of a function f to a *context* (a world-time-individual triple), yielding the assumptions made by the individual at that time and world.⁹⁰ It would therefore make sense to say that A' is *at* Γ just in case $A' = f(c)$ and $\Gamma \in c$. Anyway, this reformulation of the traditional definition is just making explicit the condition required for the right-hand side of my bi-conditional for *metaphysical* modal claims to hold:

$$\begin{aligned}
& M_{REL}, \Gamma \Vdash_{v*l} \Box_{A'} P(\alpha) \\
& \iff \text{for all } \Delta \in G \text{ such that } \Gamma R_{A'} \Delta : M_{REL}, \Delta \Vdash_{v*l} P(\alpha) \\
& (\iff \mathbb{E}(P(\alpha)) \in A' \text{ at } \Gamma)
\end{aligned}$$

Hence, if the reductive definition of metaphysical modal terms sketched in §5.3 is to work, *both* directions of the traditional bi-conditional (re-formulated) must hold. So Fine’s claim i) needs to be affirmed, but ii) must

⁸⁹It should be noted that the details of Fine’s account differ drastically from my own. It is only by phrasing the key aspects of his account in a sufficiently general way that I am able to agree with it.

⁹⁰See §7 for further details.

be denied. This creates a problem: for it was earlier explained that I have to show that the essentialist predicate cannot be reduced to modal terms. By committing myself to the negation of a key aspect of Fine's account, I would seem to abandon the potential repair to my system. However, it should be recalled that there are *two* ways for a definition to fail to be reductive, and the possibility of *upholding the material-adequacy* of the traditional definition of 'essential property', whilst *challenging its non-circularity*, remains open. Arguments for the material-adequacy of the modal definition shall be offered in support of my denial of ii) in the following subsection.

I shall also explain my perspectives regarding iii) - v), in the course of which I shall argue for the circularity of the traditional modal definition. Endorsing iii) is useful, since it gives more information about the still mysterious essentialist predicate \mathbb{E} : individuals in different contexts may find particular answers to the question 'What is *a*?' salient, and this is a partial explanation of the essential properties they will assume to hold of an object. I say it is a *partial* explanation because iv) claims that Fine's definition of 'essential property' is not reductive. Fine presumably believes his definition to uphold the material-adequacy condition (unlike the modal definition), and hence must be assuming that it fails the non-circularity condition. Indeed, he writes that he doubts 'whether there exists any explanation of the notion in fundamentally different terms'.⁹¹ Whilst it is difficult to defend such a strong claim, it can at least be suggested that giving criteria for when something counts as an answer to the question 'What is *a*?' invokes the essentialist predicate itself. That is, suppose individual *S* in context *c* assumes that Socrates is essentially human, and *S* justifies this assumption on the grounds that this constitutes an informative answer to the question 'What is *Socrates*?' in *c*. Asking *S* to explain *why* this is a reasonable answer in the context would probably only prompt a response along the lines of: 'Well, we say what Socrates is by noting the properties that are constitutive of his essence'. If this scenario seems plausible, and more informative criteria for a claim's counting as an answer to 'What is *a*?' cannot be conceived of, then there appears to be good evidence to grant that Fine's definition of 'essential property' is circular. It must also be argued that the traditional modal definition is circular. But this is entailed by an acceptance of v). I am committed to v) by the observation that my attempted reductive project is successful if and only if the schema for metaphysical modal claims is reductive. That is, the left-hand side of the following bi-conditional must be reducible to the right-hand side:

⁹¹Fine (1994b) p.53.

$$M_{REL}, \Gamma \Vdash_{v*l} \Box_{A'} P(\alpha) \iff \mathbb{E}(P(\alpha)) \in A' \text{ at } \Gamma$$

But if this is the case, then the left side of the traditional definition cannot reduce to the right:

$$\mathbb{E}(P(\alpha)) \in A' \text{ at } \Gamma \iff M_{REL}, \Gamma \Vdash_{v*l} \Box_{A'} P(\alpha)$$

This is because I have already chosen to endorse the view that the essentialist predicate is used in a reductive definition of metaphysical modal claims, meaning the traditional definition violates the non-circularity constraint.⁹²

This completes my arguments for the irreducibility of \mathbb{E} to modal terms, which have involved endorsing aspects of the Finean account. Before leaving this section, I must defend the denial of the key fact ii) in order to uphold the material-adequacy of my proposed analysis of metaphysical modal claims. I shall suggest replies to Fine's two arguments against the material-adequacy of the traditional definition. The broader implications of the conclusions that have been reached so far will then be assessed.

0.6.3 Defending the Material-Adequacy of the Traditional Definition

Fine's objections to the standard modal definition of 'essential property' focused on cases where certain metaphysical modal necessities putatively obtain without the related essentialist claim's obtaining. In short, Fine presents cases where the right-hand side of a given formulation of the bi-conditional holds, but the corresponding left-hand side fails to hold. I will respond to such cases by arguing that the necessity concerned is not in fact *metaphysical* necessity. It is therefore incorrect to perceive the relevant modal claims as instances of the right-hand side of the schema for metaphysical modal claims. It will be useful to recall what I consider a 'metaphysical modal claim' to be: firstly, there was the somewhat hazy definition in the introduction, where a modal claim is 'metaphysical' by virtue of having an intuitively metaphysical subject matter; secondly, I may now employ the proposed truth-conditions and hold that a *true* modal claim is 'metaphysical' if and only if either its analysis, *or* an analysis of its negation that

⁹²I am taking it for granted that definition is a matter of choice, though one must weigh up the theoretical considerations in favour of reductively defining t in terms of t' versus reductively defining t' in terms of t . I shall weigh up such theoretical considerations towards the end of this section.

assumes its truth, reveals an assumption-set that contains an essentialist predicate.⁹³

It might be objected that this response to Fine’s two cases renders the material-adequacy of the traditional modal definition unfalsifiable, since I may respond to any accusation of a mismatch between the truth-values of the left and right sides by concluding that the schema is in fact being misapplied. However, I think this objection is side-stepped if I am capable of giving an informative analysis of Fine’s putative metaphysical necessities in terms of assumption-sets that lack an essentialist predicate; for, after all, the attempts of doing without the essentialist predicate in analyses of the irrefutably metaphysical claim ‘It is possible for Adam to have been a snake’ in §4.1-§4.2 were a resounding failure. I shall therefore proceed with the described strategy.

Fine’s first objection to the traditional definition was that ‘Belonging to singleton Socrates if Socrates exists’ counter-intuitively becomes an essential property of Socrates, due to the fact that it holds of him in every world accessible in a certain context. One way to formalise the supposed metaphysical modal necessity (assuming we may quantify over sets) would be:

$$M_{REL}, \Gamma \Vdash_{v*l} \Box_{A'} (\exists s)(\forall y)(y \in s \leftrightarrow y = socrates)$$

The question to be determined is whether this is a metaphysical necessity. Using the two criteria noted above, I shall reach a negative answer to this question. Firstly, Fine himself denies that the necessity of the existence of the singleton Socrates derives purely from Socrates’ nature, and I would agree; however, I am inclined to use this intuition to bolster a denial that the

⁹³The initial temptation was to say: ‘a modal claim is ‘metaphysical’ if and only if its analysis reveals an assumption set that contains an essentialist predicate’. However, this definition would not suffice to identify a haecceitist’s issuing of the claim ‘it is not possible for Adam to have been a snake’, which she takes to be *false*, as ‘metaphysical’; that is, such a claim is false only if the relevant assumption set *lacks* any essentialist claim. We would need to consider what assumptions are required to render this claim *true* in order to categorise it as ‘metaphysical’ on the basis of the presence of an essentialist claim. However, it is not sufficient to say ‘a *true* modal claim is ‘metaphysical’ if and only if its analysis reveals an assumption set that contains an essentialist predicate’. For a haecceitist’s claim that ‘it is possible for Adam to have been a snake’ is true only if his assumption-set *lacks* an essentialist claim concerning Adam’s being human. Yet we can guarantee that the negation of this claim, ‘it is *not* possible for Adam to have been a snake’, will be true only relative to an assumption-set that includes essentialist claims about Adam. This is why identifying metaphysical modal claims on the basis of the assumption of essentialist claims can only be expressed in such an unwieldy manner.

topic of this claim is ‘metaphysical’, rather than to challenge the traditional definition of ‘essential property’. Secondly, I do not think that an essentialist predicate unavoidably appears in A' , which means I can attribute the necessity of the modal claim to another flavour of modality. It seems that Socrates’ existence should entail the existence of the singleton Socrates because it is a consequence of the *axiom of pairing* from Zermelo-Fraenkel set theory that the singleton set $\{a\}$ exists for any object a .⁹⁴ Yet the axiom of pairing can obviously be stated without making use of an essentialist predicate. It therefore seems that the modal claim concerns what we might refer to as *mathematical* necessity, since certain mathematical axioms may be added to an empty assumption-set in order to render the relevant modal claim true.

Fine’s second objection to the modal definition of ‘essential property’ is that every absolute necessity (i.e. those truths that hold in every world) becomes an essential property of any given object, since the truth holds in every world in which the object exists. We may formulate the sort of claim Fine has in mind by creating a predicate that reflects such a necessary truth in order to apply it to Socrates. For example, Fine’s second criticism follows from the metaphysical necessity of a claim such as:

$M_{REL}, \Gamma \Vdash_{v*l} \Box_{A'} \textit{Exists-in-a-world-in-which-} \neg(\exists y)(P(y) \wedge \neg P(y)) \textit{-is-true (socrates)}$

or:

$M_{REL}, \Gamma \Vdash_{v*l} \Box_{A'} P\text{-or-not-}P(\textit{socrates})$

Yet both versions of this claim are true relative to an empty assumption set A' . Moreover, the topic of this claim does not strike me as particularly metaphysical. Once more, then, this is not a *metaphysical* modal claim.

In short, using assumption-relative modal operators means that the option of denying that a particular modal claim is metaphysical can always be pursued, in order to explain the intuitive falsity of the left side of the relevant bi-conditional. Provided an assumption-set can be specified that renders the appropriate modal claim true without the presence of an essentialist predicate, the applicability of the essentialist bi-conditional may be challenged. For such cases, Fine is correct that the relevant essentialist claim does not hold, but is incorrect to expect it to be predicted to hold.

⁹⁴Leung and Chen (1992) p.30.

Finally, it is clear that this strategy cannot be successfully implemented for *every* modal claim, since certain claims can only be analysed through the use of an assumption-set with an essentialist predicate; *these are the metaphysical modal claims*. This completes my argument, contra Fine, for the material-adequacy of (my formulation of) the traditional modal definition of ‘essential property’.

0.6.4 The Unwelcome Consequences of Taking \mathbb{E} as Primitive

In this section, it has been seen that true modal claims with a metaphysical flavour may be characterised in terms of their relativisation (or the relativisation of their true negations) to assumption-sets that contain the essentialist predicate. It was argued that this essentialist predicate may be understood without invoking modal terms, and that it is itself used in the reductive definition of modal terms. The essentialist predicate must therefore be endorsed as a primitive.

However, we may now return to the conclusions of §2.3 and ask the question of whether this definition increases the likelihood that one will commit oneself (in the comparative sense) to fewer primitives than any rival. It was argued that this question was to be answered affirmatively by providing a reductive definition of modal terms before establishing that this definition uses non-modal primitive terms that are difficult to reductively define. A non-modal primitive term was said to be difficult to reductively define if either no combination of terms can be found that provide a reductive definition of it, or if it appears to have a good claim to being a plausible primitive; and evidence for its being difficult to reductively define was said to emerge if the majority of definers treat it as a primitive.

Let us take stock, then. Firstly, §5.1-§6.3 demonstrated that a reductive account of metaphysical modal claims is available. To determine whether this account succeeds in increasing the likelihood of minimising the primitives we (comparatively) commit ourselves to, the difficulty associated with reductively defining the primitives that we (non-comparatively) commit ourselves to must be ascertained. These primitives were said to be *set*, *sentence* and \mathbb{E} , with the two former already argued to be difficult to reductively define in §3.3. To argue for a similar conclusion with regards to the latter, it must be shown that either no reductive account of \mathbb{E} is forthcoming, or that \mathbb{E} is a plausible primitive. In §6.1, the traditional definition of ‘essential’ was described, which gives a reductive definition by employing modal terms. This means that the most clear evidence for a term’s being difficult to reductively define is absent (and is in fact contradicted) in this case,

since the majority of definers have until recently treated ‘essential’ modally. One criterion for \mathbb{E} ’s being difficult to reductively define is thus violated, since some combination of terms has been found that allow a reductive definition to be given. Everything rests on the second criterion, then. Fine clearly thinks that ‘essential’ is a plausible primitive, and it might therefore be argued that \mathbb{E} meets the criteria for being difficult to reductively define on the grounds that the second disjunct holds. However, the subjectivity of the principle of picking plausible primitives now returns with full force. For advocates of the traditional definition of ‘essential’ presumably endorse the view that modal terms are difficult to reductively define by virtue of being plausible primitives, in conflict with the intuitions I confessed to in §2.1. The answer to the question of whether a reductive definition of modal terms increases the likelihood of limiting the primitives we commit ourselves to therefore rests entirely on a criterion which some definers would argue in fact supports a *reductive* definition of \mathbb{E} that employs modal terms as *primitives*.

Eschewing talk of the *likelihood* of limiting primitives, let us instead compare two particular rival definitions in order to count the primitives that they (comparatively) commit their definers to. My reductive definition of ‘necessary’⁹⁵ commits me (non-comparatively) to the primitives *set*, *sentence* and \mathbb{E} . The primitivist definition of ‘necessary’ (non-comparatively) commits its definer to a sole primitive. If we continue to assume that *set* and *sentence* are generally taken as primitives, then the primitivist about modality who endorses the traditional definition of ‘essential’ (comparatively) commits himself to the primitives *set*, *sentence* and *necessary*. I am also (comparatively) committed to three terms. Disappointingly, the limiting primitives principle therefore does not adjudicate between the rival accounts.

I would diagnose the situation as follows: individuals who are attempting a reductive definition of a term t will frequently encounter an unwelcome primitive t' in the course of addressing the complexities required to render their definition functional. This primitive, which was not anticipated when the reductive project was initially envisaged, causes one’s (comparative) commitments to sharply rise relative to a definition that employs a reductive account of the unwelcome primitive. This means that a project that involves a reductive definition that initially appeared likely to minimise the number of primitives one (comparatively) committed oneself to ends up faring no better than a rival account according to the principle of limiting primitives.

⁹⁵It obviously does not matter which modal term I take as basic, since they are both definable in terms of the other.

It has been concluded that there is no objective criterion that will adjudicate between a reductive and a primitive definition of ‘necessary’ on the basis of the number of primitive terms that the definers commit themselves to. Yet I would argue that this does not nullify the account that has been described: firstly, it should be noted that the current reductive account is no worse than a primitive definition of modal terms with respect to the primitives it commits an individual to. In this sense, I have correctly followed the recommendations of the principle of limiting primitives (as has the modal primitivist who gives a reductive modal definition of ‘essential’, seemingly). Secondly, the subjective criterion of picking plausible primitives might cause an individual to endorse a reductive account of modal terms over a primitive one, on the basis of skepticism about the plausibility of taking the term ‘necessary’ to be immediately understandable. Thirdly, certain theoretical considerations might weigh in favour of the current account. These could include the fact that the space of possible worlds is defined with total clarity, that a uniform approach to all modal claims has been sketched or that the account appears to accurately reflect what is meant by modal claims. I therefore wish to conclude that this account has provided an analysis of the meanings and truth-values of metaphysical claims in line with the aims set out in the introduction; however, objective motivation for this definition *qua reductive* account is absent.

0.7 Objections and Replies

There are several objections to the developed account that may well have become apparent throughout the previous sections. My aim in this final section is to sketch several available responses to these objections. In many of my replies, I shall refrain from committing myself to any particular solution. There are three reasons for this aversion to commitment. Firstly, my aim continues to be to produce a flexible system that is neutral with regards to all matters that I can avoid taking a specific perspective on. Secondly, space limitations mean that I cannot give detailed arguments in support of particular positions, hence I could not fully justify an endorsement of a position. Finally, many of the objections can be addressed merely by showing that several options that are compatible with my account exist, therefore endorsing one specific option is not required. Having emphasised these important qualifications regarding my intended aims within this section, I shall proceed with my responses to the most pressing objections.

1. *This account applies only to simple modal predicate logic expressions. Can this picture be extended to the modal claims issued by actual individuals in a more complex natural language?*

The motivation for attempting to make sense of simple translations of modal claims was to avoid the complexities afforded by natural language syntax. Theoretically, there is no reason that the developed account could not be adapted for (say) modal claims in English. Yet there are practical limitations: the current absence of a grammar that generates all and only the well-formed strings of English precludes the construction of sets that are *maximal* with regards to the sentences of the language. This is why semanticists working on modality would be reluctant to endorse reductive accounts such as the current one.

However, it is also worth noting that the current account bears a heavy debt to semanticists' work on natural language modals. Semanticists do not consider any natural language modal claims to be 'absolute', insofar as they hold that certain assumptions are operative in each context to restrict the accessible worlds.⁹⁶ Hence, whilst attempting to adapt the reductive aspect of this account to natural languages may result in practical issues, the non-absolutist aspect of the account is already acknowledged to be a vital feature of any treatment of natural language modal claims.

2. *The account entails several counter-intuitive theses about the process of issuing modal claims that contradict what we take ourselves to be doing in issuing such claims. This undermines the account.*

In order to address this objection, it makes sense to begin by precisely categorising the account. Once the most important features have been extracted, I shall identify which features appear to accord with what we take ourselves to be doing in issuing modal claims. I will then discuss how to handle the aspects of the account that do not seem to accord with modal discourse.

The account falls under the following categories:

1. *Modal reductivism* - Modal terms can be defined through non-modal terms. Specifically, I employ the primitives 'set', 'sentence' and 'essential' in my reductive definition. This view contrasts with modal

⁹⁶See Kratzer (1991)

primitivism, where modal terms are themselves taken as primitives.

2. *Modal monism* - there is a ‘single, underlying modal notion in terms of which all others could be defined or understood.’⁹⁷ Specifically, logical possibility is used to define an absolute space of worlds, with modal claims that express other flavours of modality characterised in terms of subsets of this absolute space. This contrasts with modal pluralism, where different flavours of modality are interpreted with respect to disjoint spaces of worlds.
3. *Modal factualism* - modal claims have truth values. This contrasts with modal non-factualism, whereby the meanings of modal expressions are given in non-truth-conditional terms.
4. *Modal non-absolutism* - modal expressions are always interpreted ‘relative to contextually determined sets of background assumptions’.⁹⁸ This is in contrast to modal absolutism, which holds that the truth-values of modal claims are generally assigned in such a way that only the world of utterance affects them.

I endorsed modal reductivism on the basis of a general principle that recommends adopting available reductive accounts, in addition to a personal intuition that modal terms are not plausible primitives to endorse. I would not predict that natural language speakers have intuitions regarding the plausibility of primitive analyses of modal terms whilst immersed in the process of issuing modal claims. I would therefore deny that modal reductivism is at odds with speakers’ experiences of modal discourse.

My commitment to modal monism derived from the fact that it is the most viable way of implementing a reductive account. In short, giving a reductive definition of all modal terms by employing the same adaptable apparatus is a more simple strategy than giving distinct reductions of modal terms that express different flavours of modality. Once more, I do not think that speakers would tend to have experiences that either accord with or contradict modal monism, hence I would suggest that speakers’ intuitions are neutral with regards to modal monism.

Modal reductivism and modal monism do not appear to entail any predictions about modal discourse that speakers’ experiences could confirm or disconfirm, which explains why I would not categorise these aspects of my

⁹⁷Fine (2002) p.254.

⁹⁸Lycan (1994) p.171.

account as features that fail to accord with speakers' intuitions. However, the matter is less clear-cut for the other two aspects of my account.

Modal factualism was a position I arrived at through tacitly accepting the assumptions of modal logic and standard formal semantic accounts of modal terms, within which expressions such as ' $\Diamond\phi$ ' and ' ϕ is possible' are assigned truth-conditions and truth-values. It is also the case that the reductive account I arrived at involved a bi-valent means of rating each modal claim, depending on membership or non-membership of the non-modal prejacent in the relevant maximal-consistent sets. Bi-valent systems are obviously naturally extendible to truth-conditional systems. A modal factualist account makes clear predictions that speakers' experiences may or may not reflect: that is, modal claims will be truth-apt. If it is granted that speakers have differing intuitions about, and responses to, truth-evaluable expressions and non-truth-evaluable ones, then factualist accounts of modal discourse may be supported or challenged by speakers' experiences. The most well-developed non-factualist accounts treat modal terms as *force-modifiers*, meaning that prefixing a non-modal claim with a modal term affects the force with which the non-modal claim is asserted without causing a new proposition to be issued. I shall therefore predict and compare the intuitions evoked by: 'It's possible that it is raining' (an epistemic modal claim) to those evoked by: 'Speaking frankly, she's too good for him'⁹⁹ (an embedded truth-evaluable claim prefixed by what is clearly a force-modifier). The test assertions will involve embedding the two claims in expressions for which it is generally accepted that only claims with truth-values will sound felicitous:

1. 'If it's possible that it's raining, we should bring umbrellas'
2. 'It's not the case that it's possible that it's raining'
3. 'Red believes that it's possible that it's raining'
4. 'If speaking frankly she's too good for him, then she should leave him'
5. 'It's not the case that, speaking frankly she's too good for him'
6. 'Red believes that speaking frankly she's too good for him'¹⁰⁰

I predict that a speaker would find the first three comments to be acceptable, and would find the last three unacceptable. Of course, the non-factualist could attempt to explain away the above data. However, I only needed to

⁹⁹MacFarlane (2011) p.156.

¹⁰⁰*Ibid.* p.157.

show that speakers' experiences of modal discourse *do not contradict* factualism, since I am not trying to *argue for* modal factualism on the basis of intuitions. Provided my predictions of the elicited intuitions are considered plausible, I conclude that modal factualism's predictions regarding modal discourse do not conflict with speakers' experiences.

Now it remains to consider modal non-absolutism. I arrived at modal non-absolutism partly by taking seriously the accessibility relations that are always employed in modal logics and in formal semantic accounts of modality. Furthermore, it was already mentioned that the simplest way to implement modal reductivism is to employ a single space of worlds (modal monism) before singling out different subsets in order to deal with modal terms of diverse flavours. If these subsets are selected by contexts within the actual world, rather than depending only on the world of utterance, then modal non-absolutism results. Like modal factualism, non-absolutism makes clear predictions that speakers' experience may or may not reflect: different tokenings of the same modal claim may take different truth-values (even if we appropriately alter other indexical terms within such claims), since they will turn out to employ different assumption-relative accessibility relations under analysis. This prediction may therefore be tested.

It should be clear that modal absolutism is implausible as a general thesis about modal discourse. For it is difficult to see how one could deny that a modal claim such as 'It's not possible for me to go to the party' can express different flavours of modality in different contexts, and may take different truth-values even if the flavour is held fixed. However, this is not to rule out a version of modal absolutism that occurs only with respect to a subset of the different flavours of modality. Most plausibly, a selective absolutist would endorse modal absolutism for *metaphysical* modal claims alone. Linguists are often doubtful that the modal claims expressible in natural language include a class of *metaphysical* modal claims;¹⁰¹ hence an absolutist about metaphysical modality would be likely to consider the modal discourse engaged in by *metaphysicians* to be what challenges the predictions of modal non-absolutism. And this is the point at which I concede that the current theory runs counter to the experiences of those immersed in modal discourse: *the non-absolutist aspect of my account contradicts what metaphysicians take themselves to be doing when engaged in metaphysical modal discourse.*

There are two ways in which one might attempt to uphold the current account in light of this divergence from metaphysicians' experiences of issuing modal claims: one might either present some arguments in support

¹⁰¹Portner (2009) p.133, footnote 2.

of the idea that metaphysicians who take themselves to be issuing modal claims with absolute truth-values are just wrong about what they are doing; or one might leave open the potential for a particular set of assumptions to be the ‘correct’ set of assumptions for metaphysical modal claims, and say that the metaphysical modal claims that are true relative to this privileged set of assumptions hold in a more secure way than other metaphysical modal claims do. I shall proceed with sketching both strategies.

Firstly, one could challenge the intuitions that metaphysicians have about the modal claims they make. One might say that, given that either all or most¹⁰² modal claims expressed in natural language are assigned truth-values relative to restricted sets of possible worlds, it is less likely that metaphysical modal claims are special than that metaphysicians are mistaken about the truth-values of their modal claims. However, this line of argument is unlikely to impress metaphysicians, since they may well be convinced that metaphysical modal claims *are* special. Hence, even if this first line of argument is compelling for non-metaphysicians, it clearly would not succeed in easing the metaphysician’s concerns about the non-absolutist aspects of my account.

A second strategy may therefore be pursued as follows: if the metaphysician maintains that her modal claims take absolute truth-values, then we might ask which space of worlds it is that their truth-values are invariably determined relative to.¹⁰³ This space would obviously not be the absolute space defined within my account, since it has already been pointed out that alleged metaphysical impossibilities do not gain their status by virtue of expressing logical contradictions. The putative invariant space of worlds for metaphysical modal claims therefore is not co-extensive with my absolute space. This space is presumably not broader than my absolute space, unless logical contradictions are metaphysically possible. Hence the space of metaphysically-accessible worlds must be a subset of my absolute space of worlds. Yet there is great flexibility over the subsets of the absolute space that the assumption-relative accessibility relations of my system allow to be selected.¹⁰⁴ Therefore, the metaphysician who endorses modal absolutism

¹⁰²Depending on whether natural language modals can express logical possibility.

¹⁰³Assuming that the metaphysician is expecting *all* metaphysical modal claims to be analysed with respect to the same space, as opposed to merely expecting that all *tokenings* of a particular metaphysical modal claim will be analysed by means of the same space.

¹⁰⁴Though note that it is not the case that my system allows *any* subset of worlds to be selected. To illustrate this point, consider the subset of the absolute space that consists of only Γ and Δ , where for each $\phi \in \Gamma$, $\neg\phi \in \Delta$ (and vice versa). No finite, consistent set of assumptions A would render this subset accessible, since for each $\psi \in A$ it cannot be that $\psi \cup \Gamma$ is consistent *and* that $\psi \cup \Delta$ is consistent. An empty assumption-set would

should have no quarrel with the formal machinery of my account. Rather than making use of an alternative system that gives a pre-determined space of metaphysically-accessible worlds, he need only identify the privileged set of assumptions that yield the invariant set of metaphysically-accessible worlds within my system. In such a scenario, the stipulation that this set is the correct one for the interpretation of modal terms expressing a metaphysical flavour could be added to my account. Then, any individual who issues a metaphysical modal claim relative to a different assumption-set has somehow erred. I must confess to doubts about the viability of determining a set of objective essential properties for an individual, and subsequently about the plausibility of then postulating the existence of such a set; and such doubts are precisely why I have described a system that accommodates the idea that there is no such privileged set of assumptions for metaphysical modal claims. Of course, my skepticism might prove to be unfounded, and compelling arguments may be given by modal absolutists for a particular set of essentialist claims' being the privileged set.

In summary, I have argued that neither the reductive, monist nor factu-alist aspects of my account clash with natural language speakers' intuitions about modal discourse. The non-absolutist aspect also most likely accords with speakers' experiences of modal claims in general, whilst perhaps contradicting the experience of metaphysicians in metaphysical modal discourse. I am open to the view that such experiences should be deemed mistaken, but a more convincing argument for metaphysicians holds that absolutism about metaphysical modal claims is compatible with my account.

3. The account suggests that assumption-sets are personal choices. But surely there are some constraints that need to be imposed.

Holding that assumption-sets are fixed through unconstrained personal choice implies that situations may occur in which an individual assumes that 'bachelor' means 'man', and thereby derives the truth of 'it is possible for there to be an unmarried bachelor'. This concern about *strange postulates* ('strange' in the sense of deviating from the sort of things that the majority of individuals might assume in similar contexts) indicates that more needs to be said about the content of assumption-sets. Another concern that should be

also not render just this subset accessible, since the absolute space will be accessible in such a scenario. However, aside from this limitation, it is clear that assumption-sets can be formed that allow a diverse array of subsets of the absolute space to be selected.

addressed involves *irrelevant assumptions*. That is, it seems preferable to deny that an individual immersed in a discussion about bachelors assumes anything about Adam’s essential properties, since we would otherwise leave open the potential for assumption-sets’ being so large that it becomes implausible to postulate that an individual bears a relationship (whether the relationship consists of representing that set or something else) to such a set. In order to deal with both concerns, I shall sketch some constraints on the content of assumption-sets.

I am going to suggest that an assumption-set consists only of an agent’s *beliefs* about items related to the *question under discussion*. Of course, there is a lot of debate within philosophy about the nature of beliefs, but I need only assume that they have propositional content which is formulable within the language of elements of assumption-sets. The term ‘question under discussion’ refers to the question that is explicitly or implicitly being discussed in the conversational context.¹⁰⁵ This notion derives from an area of semantics in which it is held that ‘discourses are structured around questions. It would of course be maximally useful to have an answer to the Big Question —What is the Way Things Are?— but this question is rather unmanageable. Instead, speakers adopt strategies for inquiry that involve dividing the Big Question into smaller questions which are more tractable and more directly relevant to current purposes.’¹⁰⁶ Let us say that the *items* related to the question under discussion include all terms within the question under discussion, all synonyms of those terms,¹⁰⁷ and all objects referred to by the terms or their synonyms.

This addresses the concern about irrelevant assumptions by entailing that an agent will stand in a relation to a fairly tractable assumption-set, since only her beliefs about the terms employed and objects alluded to in the topic under discussion shall enter into the assumption-set that constrains modal claims within that context. A consequence of this is that, if the question under discussion concerns Adam’s being a snake, then meaning postulates about bachelors will not be relevant; hence, amongst the accessible worlds that are compatible with assumptions concerning Adam and snakes, there will be worlds in which married bachelors exist. If one considers this problematic, then one might stipulate that there are certain assumptions that are relevant whatever question is being discussed. Recall how it was mentioned in §4.4 that I would have no complaint about a set of ever-present

¹⁰⁵See Roberts (1996).

¹⁰⁶Lassiter (2011), p115.

¹⁰⁷See §2.3 for a rough characterisation of synonymy in terms of the ordering relation an individual uses to define the terms of their language.

assumptions' being specified. As earlier noted, generating the particular assumptions for inclusion in this set will be a significant challenge. It will also be a difficult balancing act to ensure that the set of ever-present assumptions is sufficiently large to rule out unwelcome worlds at the same time as being sufficiently small as to be tractable. The presence of these two challenges might provide motivation to deny the existence of any ever-present assumptions, thereby conceding that there will be some accessible worlds that represent bizarre facts. Such bizarre facts would be unrelated to the topic under consideration so could be safely ignored. My preference would be to accept this motivation and avoid the difficulties associated with postulating ever-present assumptions; though either approach could be adopted to render assumption-sets finite.

The concern about strange postulates is addressed as follows: if an agent is under the mistaken impression that 'bachelor' means 'man', then this would be the only meaning postulate about the term 'bachelor' that would appear in his assumption-set when the topic under discussion involves bachelors. Once it becomes apparent to other individuals engaged in the conversation that the agent is making such an assumption (for example, if the agent says 'It's possible for there to be married bachelors', which would be *true* relative to her assumption-set), they might point out that he is assuming something that they consider strange. On the other hand, an individual who believes that 'bachelor' means 'unmarried man' may not omit this assumption when it is relevant to the topic that is being discussed. If an individual who knows perfectly well what 'bachelor' is generally taken to mean insists on telling someone 'It's possible for there to be married bachelors' out of perversity, then he would be saying something that is *false* relative to his assumption-set. In short, individuals make strange assumptions if that is what they genuinely believe, and certain strange modal claims will then be true relative to those assumptions. There may be ways to convince agents that their assumptions are strange, and it could perhaps be said that modal claims which are true relative to strange assumption-sets possess a defect lacked by modal claims which are true relative to less strange assumption-sets. Yet an agent is barred from making assumptions that they believe to be false, other than in the context of a counterfactual conditional.¹⁰⁸

In conclusion, I am able to say a little more about the content of assumption-sets in order to ease concerns about strange postulates and irrelevant as-

¹⁰⁸I would argue it is the very fact that individuals normally assume things they take to be true that leads to the requirement to explicitly state counterfactual assumptions in the form of antecedents (e.g. 'If bachelor' meant 'man', then it would be possible for there to be married bachelors').

sumptions. This has been a very rough sketch of the relation between assumption-sets, beliefs and questions under discussion; and a more detailed treatment of the topic may or may not reveal problems with the approach.

4. When one agent issues a modal claim and another agent considers it, just whose assumption-set is the second agent employing?

As of yet, I have not explained how relative accessibility relations are selected in particular contexts. This means that, although my account predicts the truth-value of a modal claim relative to specific assumption-sets, it does not predict which assumption-set is operative in a given context. I need to define a function f from contexts (understood as world, time, agent triples) to the assumption-sets operative in those contexts. This function would be triggered by the occurrence of a modal term at that context (which has been signified in the preceding account through prefixing the modal terms by a subscript referring to the assumption-set that they are relative to). The set of accessible worlds would then be generated by applying a relative accessibility function to that relevant assumption-set, in order to yield a relative accessibility relation. Yet even after defining this function f , it does not automatically follow from the fact that a modal claim is uttered at a context c that the relative accessibility relation relevant to its interpretation is generated by applying f to c . It remains to be determined whether my assumption-sets should be generated within a contextualist framework, a relativist framework, or another one altogether. I shall begin by summarising the debate between the proponents of the traditional contextualism and the more recent relativism with regards to epistemic modals, before tentatively taking a position on the matter.

The traditional approach within formal semantics is to emphasise the *context of utterance* (a world, time, individual triple at which a claim is issued) as the determiner of the space of accessible worlds.¹⁰⁹ That is, for an epistemic modal claim, the accessible worlds are those compatible with the knowledge of some individual or group at the context in which the claim is issued; for a deontic modal claim, the accessible worlds are those compatible with the relevant rules operative at the context of utterance, and so on. On this view, a modal claim expresses a particular proposition once it is uttered in a context, since an assumption-set relevant to the claim's interpretation

¹⁰⁹For example, see Kratzer (1991)

must be identified for it to possess content.¹¹⁰

Yet it has been argued (for example, in MacFarlane (2011)) that the entire spectrum of contextualist views demonstrably fail to predict the correct truth-conditions for epistemic modals. The data used to motivate this perspective typically consists of either ‘eavesdropping’, ‘retraction’ or ‘disagreement’ cases. The following is an example of an ‘eavesdropping’ case: suppose Prim is looking for Red, and Prim says to a conveniently located by-stander ‘It’s possible that Red’s outside’. Red, who is hiding behind the curtains, overhears this conversation and mutters to herself ‘What Prim said is false’. Someone who is wanting to argue against contextualism hopes that those who hear this scenario will have two intuitions about the case: firstly, that Prim spoke appropriately in uttering the modal claim and, secondly, that Red also spoke appropriately in her judgement of the claim as false. It is then argued that contextualists cannot accommodate both intuitions. This is the case because a version of contextualism that takes Prim’s knowledge to determine the accessible worlds relevant to the interpretation of her claim cannot accommodate the second putative intuition; for Red’s claim is neither appropriate in the strong sense of being true (for since it is true that Red’s being outside is compatible with Prim’s knowledge, Prim’s modal claim is true, so Red’s assertion that Prim’s claim is false must itself be false), nor in the weak sense of being a justified utterance in the context (since an eavesdropper seemingly has no business commenting on the knowledge of another individual, since she is not immersed in a conversation with that individual). Moreover, a version of contextualism that takes Prim and Red’s knowledge to determine the accessible worlds relevant to the interpretation of Prim’s claim (on the grounds that Red’s presence, even unbeknownst to Prim, makes her a participant in the conversation) cannot accommodate the first putative intuition; for Prim’s claim is then neither appropriate in the strong sense (since it is false that it is compatible with both Prim and Red’s knowledge that Red is outside) nor in the weak sense (since Prim is surely not justified in speculating about the knowledge states of all individuals and potential eavesdroppers in the vicinity).¹¹¹

Relativist perspectives are inspired by taking the intuitions evoked by ‘eavesdropping’ cases seriously, at the same time as assuming the strong interpretation of appropriateness. This leads to an account according to

¹¹⁰I have been trying to avoid talk of propositions in the preceding sections, in order to simplify matters. Yet it is important to resort to such talk now in order to clarify the difference between contextualism and relativism. I shall simply remain neutral about what propositions are.

¹¹¹MacFarlane (2011) p.155

which Prim’s modal claim expresses a single proposition which is both *true-for-Prim* and *false-for-Red*.¹¹² Where contextualism assumed that tokenings of modal claims take a single truth-value at a world, fixed by factors associated with the context of utterance, relativism therefore allows a single utterance of a modal sentence to take diverse truth-values within a single world relative to different *contexts of assessment* (world, time, individual triples at which a claim is judged).¹¹³ Standard relativism holds that it is always the assumptions of the individual who is part of a particular context of assessment that determine the interpretation of the modal claim at that context. Relativists hold that Red’s muttering ‘What Prim said is false’ is appropriate (in both senses) because what Prim said is indeed false, relative to Red’s knowledge at the world and time at which she assesses the claim; and Prim’s initial utterance is appropriate because it is true when assessed relative to the context in which she utters it.

This is a highly complex area of debate. In fact, recent experimental data suggests that individuals’ semantic intuitions about epistemic modal discourse do not fit neatly within either a contextualist *or* relativist framework.¹¹⁴ My inclination is therefore to avoid committing myself to any position on the matter, at least until further data about speakers’ semantic intuitions surrounding modal discourse is collected. However, I am aware that this results in my account’s failing to generate any predictions whatsoever about the truth-values of particular modal utterances. In light of this fact, I shall very tentatively endorse a position on the matter. It is preferable to select the most flexible position available, in order to allow my account to accommodate the unpredictable semantic intuitions evoked by even the most creative ‘eavesdropping’, ‘retraction’ and ‘disagreement’ cases. With this aim in mind, I will tentatively endorse *flexible relativism*. For epistemic modals, this view holds that ‘whose knowledge is relevant to the evaluation of epistemic modals is *itself* determined by features of the context of assessment’.¹¹⁵ Thus it is being suggested that, for all modal terms that occur without an explicit restricting phrase, pragmatic factors surrounding the context of assessment determine the assumption-set that the relative accessibility function is applied to. Flexible relativism may agree with standard relativism that, in general, an individual at a context of assessment selects

¹¹²This is obviously a special sort of proposition, since propositions are not normally construed as entities that may be assigned different truth-values at different contexts of assessment.

¹¹³MacFarlane (2011) p.159.

¹¹⁴Knobe and Yalcin (2013) pp.15-6

¹¹⁵MacFarlane (2011) pp. 175-6.

an assumption-set consisting of their own beliefs relevant to the topic at hand to interpret a modal claim. However, flexible relativism permits the individual at a context of assessment to occasionally select someone else's assumption-set, or the assumption-set consisting of the union of her own relevant beliefs and someone else's, in order to fix the space of accessible worlds. This would presumably be triggered by the question under discussion at that individual's context of assessment. For example, if the question that is implicitly being discussed at Red's context of assessment is whether Prim was justified in issuing her initial utterance of 'It's possible that it's raining', then Red might select Prim's own assumption-set at Prim's context of utterance to assess the modal claim, to the best of her abilities.¹¹⁶ Of course, in most scenarios the implicit question under discussion would be said to consist of the *truth* of Prim's initial utterance of 'It's possible that it's raining', in which case Red will employ her own assumptions against which to test Prim's claim.

A limitation often noted for flexible relativism is that it makes minimal predictions for the truth-values of modal claims relative to particular context of assessment. However, it is certainly preferable for me to employ an account of assumption-set selection that makes *minimal* predictions than for me to refrain from employing an account at all, thereby making *no* predictions for the truth-values of modal claims. Whilst the flexibility of flexible relativism means that the truth-values assigned to modal claims at particular contexts of assessment cannot reliably be predicted, this is a normal feature of accounts of linguistic phenomena that postulate a significant role for pragmatics. Moreover, this flexibility is precisely what accommodates the unexpected intuitions that have emerged from recent data, since flexible relativism anticipates that speakers will be unable to reliably discern from an abstract consideration of a case the assumption-set that is fixed by the context of assessment within that particular case. Lastly, I shall reiterate that my endorsement of any position on this complex matter is reluctant, and the general account described throughout the previous sections may be adapted for any version of contextualism or relativism should compelling data arise in support of that version.

¹¹⁶It has already been shown that relativists partially motivate their position by pointing out the implausibility of individuals' reliably utilising other people's assumption-sets, due to the inaccessibility of others' mental states (i.e. recall that it was said that neither Prim nor Red should be expected to employ each others' knowledge when issuing or assessing epistemic modal claims in the 'eavesdropping' case). This is why the flexible relativist must say that the individual at a context of assessment would utilise someone else's assumption-set only rarely, and that she does this only to the best of her abilities.

5. Relativist theories of metaphysical modal discourse are unwarranted, since the type of ‘eavesdropping’ cases that typically motivate relativism with regards to epistemic modal claims do not sound compelling when constructed for metaphysical modal claims.

For me, the motivation for endorsing relativism for metaphysical modal claims would not derive from a consideration of ‘eavesdropping’, ‘disagreement’ and ‘retraction’ cases constructed with metaphysical modal claims. Rather, my account predicts that the flavour of modal claims is determined by the type of assumptions included in the assumption-set. Hence the context that determines the relevant assumption-set also determines the flavour of modality. Therefore, if relativism is endorsed for some flavour of modal claims, it must be endorsed for all flavours. In other words, it would not make sense to hold that an epistemic modal claim has the relevant assumptions determined by contexts of assessment whereas a metaphysical modal claim has the relevant assumptions determined by the context of utterance. This is the case because the flavour of the modal claim is not itself decided until the assumption-set is fixed.

Yet it might be argued that, if a consideration of ‘eavesdropping’ scenarios reveals the predictions of flexible relativism to be unintuitive, then this would count against the current proposal. For several reasons, I think it is difficult to argue that the relativist’s prediction clashes with the intuitions one would expect a natural language speaker to have about such scenarios: firstly, if one were to claim that there *is* something deviant about the relativist’s reading of the case, then this sense of deviance could be attributed to the rarity of such metaphysical modal claims in everyday discourse. In short, the fact that such claims are not usually discussed outside of philosophy departments might cause the mere description of this case to seem ‘weird’, resulting in a sense that *any* predictions of the semantic intuitions evoked by the case are inaccurate. Secondly, the metaphysician is free to specify some ‘correct’ assumptions about the essential features of individuals, before holding that an individual at a context of assessment should aim to deploy one of these privileged assumption-sets when considering a modal claim that they interpret as having a metaphysical flavour (at least if the question under discussion is the truth of that claim). Then, even if predictions that both Prim and Red are speaking appropriately in their rival truth-assignments to the same claim seem counter-intuitive, this could be

attributed to the fact that only one of them can be deploying the privileged assumption-set that lends their interpretations of modal claims additional security. I would therefore argue that ‘eavesdropping’ cases do not provide good evidence against relativist approaches to metaphysical modal claims.

6. *We often have a sense that a genuine disagreement over a modal claim is occurring. Does this account preserve disagreement?*

Contextualism and relativism give different accounts of disagreements within modal discourse. I shall describe the main features of each sort of account before considering whether they accord with our intuitions about modal discussions.

It is first worth clarifying what it means for something to be a ‘genuine disagreement’. This is surprisingly difficult to define,¹¹⁷ but restricting my focus to claims for which the only context-dependent term is a modal operator simplifies the process. For current purposes, I will therefore say that a ‘disagreement’ is any situation in which there are at least two participants who believe that they are attributing conflicting truth-values to the same proposition at the same time and world; and a disagreement is ‘genuine’ if they are indeed attributing conflicting truth-values to the same proposition in this way. It is difficult to give extensive criteria of individuation for propositions. Yet in the case of modal claims, it seems that analysis must at least reveal identical relative modal operators.

Cases in which one individual affirms, and the other denies, a modal claim that is interpreted relative to their mutual assumption-set are not the concern of the current discussion. The reason for this is that such cases are unproblematic for both contextualists and relativists: disagreement is genuine (in the sense that the exact same proposition is being targeted by the affirmation and denial), and only one judgement of the modal claim is accurate since one of the individuals has clearly miscalculated what is consistent with the assumption-set. The type of disagreement cases that concern us are those in which individuals are seemingly employing different assumption-sets in reaching their judgements of a modal claim. Let the following scenario be the target of the current discussion:

Prim: ‘It’s possible that it’s raining’

¹¹⁷MacFarlane (2007) pp.22-5.

Red: ‘That’s false’¹¹⁸

There are two ways in which a contextualist can interpret such an exchange in order to attempt to preserve disagreement. Firstly, the contextualist might say that the response involves Red implicitly issuing another modal claim (‘It’s not possible that it’s raining’) which is interpreted relative to the assumption-set operative at the context at which she utters it. If this assumption-set includes an assumption that it is raining, then the new modal claim uttered by Red is true, even if Prim’s initial claim was also true relative to her own assumption-set at the context of utterance. This first contextualist perspective therefore indicates that both participants in modal disputes may speak truthfully if they are using distinct assumption-sets. However, this perspective also implies that they are ultimately talking past each other, in the sense that the affirmation and denial are actually targeting modal claims that express different propositions. They are therefore not immersed in a *genuine* disagreement. Moreover, it has to postulate implicitly expressed modal claims when the form of the denial mirrors Red’s response. It might be considered implausible to suggest that asserting ‘That’s false’ is elliptical for ‘It’s not possible that it’s raining’.

The second contextualist perspective takes an identical position to the first perspective with regards to responses that include an explicitly stated modal claim. However, it rejects the notion of implicitly issued modal claims, thereby construing Red’s response differently. The response is taken at face value, and interpreted as attributing a particular truth-value to the very modal claim uttered by the Prim. This perspective therefore precludes the affirmer and denier from both speaking truthfully, since either Prim or Red is correct about the compatibility of rain with the assumption-set operative at Prim’s context of utterance. Unlike the previous contextualist view, this reading of the case indicates that both individuals are targeting the same proposition, and hence are in *genuine* disagreement. Whilst implicitly uttered modal claims are no longer postulated, it may now be seen as problematic that the related response ‘It’s not possible that it’s raining’ is treated so differently to the response ‘That’s false’. Moreover, in cases where an individual responds by saying ‘That’s false, it’s not possible that it’s raining’ or ‘That’s not possible’, it is not obvious which of the two treatments would be proposed.

¹¹⁸To assuage a concern that the ‘That’ of ‘That’s false’ might be targeting the prejacent rather than the entire modal claim, bear in mind that Red’s asserting ‘That’s false, it’s not possible that it’s raining’ would seem to be an equally appropriate response.

In contrast to the second contextualist response, relativism allows both Prim and Red to be speaking truthfully, since Prim's modal claim may well be true when considered from a context of assessment that is identical to the context in which she uttered it, whereas it will be false relative to Red's context of assessment if she employs an assumption-set that precludes it. Yet unlike the first contextualist response, a genuine disagreement is occurring, since the affirmer and denier are targeting the assessment-sensitive proposition expressed by the initial modal claim. Of course, an impression might remain that the disagreement posited by relativists is insubstantial, since it could easily be resolved if it were to be made apparent that the participants were employing different assumption-sets. Why, then, would participants in such disputes continue to disagree if they are assuming different things? MacFarlane suggests an explanation of the purpose of such disagreements, uniquely available to the relativist:

‘Assessment-sensitive expressions are designed, it seems, to foster controversy, where use-sensitive expressions preclude it. But what is the point of fostering controversy in “subjective” domains, if there is no (nonrelative) truth on which both parties can converge? Why shouldn’t we just talk about our own tastes, rather than ascribing subjective properties to the objects? Perhaps the point is to bring about agreement by leading our interlocutors into relevantly different contexts of assessment.’¹¹⁹

A relativist could therefore develop this idea in order to address the concern that modal disagreements end up looking somewhat futile.

It can thus be seen that, on one contextualist view, genuine disagreement is preserved (although if Red says ‘It’s not possible that it’s raining’ instead of ‘That’s false’, the disagreement is no longer genuine); and according to the relativist’s account, genuine disagreement occurs (whatever form Red uses to express her denial of Prim’s utterance). The choice between the two accounts, with regards to the matter of disagreement, therefore depends on whether or not it is perceived to be desirable for the participants who are in dispute over a modal claim to *both* speak truthfully. Nevertheless, it does not seem that disagreement scenarios should pose a particular problem for the current account.

¹¹⁹MacFarlane (2007) p.30.

7. *The problem of defining a ‘sentence’ of the world-making language is an insuperable objection traditionally raised against linguistic ersatzism, which undermines the current account.*

In order to address this concern, I intend to describe some available responses, in order to provide evidence that this objection is not ‘insuperable’. I shall not endorse any particular response, for the three reasons detailed in the introduction to this section.

The problem alluded to is the following: if we take ‘sentence’ to mean ‘sentence-token’ (i.e. sentences that have been or will be actually uttered or inscribed), then we have an insufficient number of sentences to express all possibilities; since, for example, ‘there is a noncountable infinity of real numbers, and we will need a sentence for each, specifying that the number in question exists; ... but there are only countably many sentence-tokens, each finite in length, so not even that part of the actual world consisting in the existence of the real numbers can be expressed by sets of sentences in the sense of tokens.’¹²⁰ We obviously cannot make use of *possible* sentence-tokens if we are seeking a non-circular account of modality. Relying on sentence-types is also problematic, for then ‘the appeal to sentence-types is too close to the appeal to abstract entities of an intensional or modal sort to conform to the non-modalist’s avowed program of reducing the modal to the nonmodal’.¹²¹ The objection is therefore that the reductivist linguistic ersatzers cannot give a coherent definition of ‘sentence’.

One option that many linguistic ersatzers have resorted to is to construct worlds from a *Lagadonian* language, whereby domain-members and properties denote themselves. ‘On one Lagadonian scheme, each object is allowed to function as its own name, and each universal as its own predicate; sentences are set-theoretic constructions out of this “vocabulary” along with logical connectives and quantifiers. Notice that on this Lagadonian account, there are words and sentences corresponding to nothing ever spoken or contemplated by a human speaker.’¹²² This approach solves the current problem because it can be guaranteed that there will be some *actual* Lagadonian sentence-token to represent every aspect of the actual world. A commonly noted objection to this response is that a Lagadonian language will only contain names for actual individuals, hence it is not clear how possibilities involving non-actual individuals would be represented. However, the fact that the current system utilises a constant-domain presents a poten-

¹²⁰Loux (1979) p.56.

¹²¹Loux (1979) pp.56-7.

¹²²Roy (1995) p.219.

tial solution to this concern, since non-concrete individuals are nevertheless domain-members at the actual world, thus their names should end up in the Lagadonian language. A second concern about employing a Lagadonian language is that a set-theoretic paradox seemingly results: ‘from the assumption that there is a maximal set of sentences, set theory yields the powerset of the set of all sentences; the language then yields a sentence corresponding to each element of that set – with the absurd result that there are more sentences than are in the set of all sentences.’¹²³ This paradox directly results from the fact that a Lagadonian language is stipulated to contain a name for every object (i.e. the object itself), and there must therefore be a name for each subset of sentences of the language. One solution would therefore be to stipulate that the Lagadonian language does not contain names for objects that are already sentences of the language, though this might harm the expressivity of the language.

An alternative response to this first problem would be to construct worlds from the sentence-tokens expressible within a logical or natural language, but define the set of all such sentences by providing a grammar for that language. For such an account, ‘expressible’ would not be a primitive modal term; for a listing of all sentence-tokens could be generated from the grammar if one had an infinite amount of time. Providing such a grammar for a natural language is clearly a significant challenge. However, a linguistic ersatzter may choose to construct worlds from translations of natural language claims within a less expressive language for which a grammar may be provided (much like the approach within the preceding sections). I would therefore argue that this problem for linguistic ersatzism is not insuperable, since multiple solutions exist.

8. Nothing has yet been said about how this account would handle generalised de re metaphysical modal claims such as ‘All humans are necessarily human’.

The essentialist predicate has been defined in such a way that it may only apply to constants, hence it is not immediately apparent how it would apply to quantified expressions in generalised *de re* metaphysical modal claims. One solution that springs to mind would be to translate expressions such as ‘human’ as an individual constant in these situations, so that essentialist claims may be formed within assumption-sets by prefixing the translation with the

¹²³Roy (1995) p.221.

essentialist predicate. These terms may then be treated as rigid constants that denote the appropriate set of individuals at a particular world and track this set across worlds. Without the appropriate essentialist assumptions, the individuals within this set will in some worlds lack the properties that serve as membership criteria in the original world. For example, we could name the set of all humans at the actual world ‘bob’. There will be some worlds in which the extension of the set ‘bob’ at that world includes individuals with the properties of goldfish and pencils and numbers. In fact, in each world in which Adam is a snake, ‘bob’ will include at least one snake in its extension. If our assumption-set A contains the essentialist claim: $\mathbb{E}(Human(bob))$ then, in every A -accessible world, the set bob would be identical with the set of all humans at that world. These ideas are clearly under-developed. A two-dimensional semantic framework would provide clarification, in order to distinguish between the set of individuals actually denoted by ‘human’ as it appears in other worlds (i.e. bob) and the set of individuals denoted by ‘human’ in those other worlds (i.e. the individuals with human properties at each world). Furthermore, I believe a nominalizing operator, which applies to predicative expressions to convert them to singular terms, could be useful.¹²⁴ Adapting the account to handle generalised *de re* metaphysical modal claims therefore does not appear to be an insuperable challenge.

9. *The account suggests that we consider an infinite number of infinite sets in determining whether or not our modal claim is true relative to our assumptions, which is implausible.*

It is first worth noting that the intractability of an infinite number of infinite possible worlds is an issue faced by anyone making use of possible world semantics (either for modal expressions or for other natural language phenomena). However, responding to a criticism by pointing out that it is not unique to the endorsed account is clearly insufficient. I shall therefore try to assuage this concern in a more informative manner.

The notion of a partial world may be defined, to allow us to claim that structures that are less unwieldy than maximal-consistent sets are represented by those individuals who are considering a modal claim. A *partial world* is a non-maximal, consistent, finite set of *wff*. Given the earlier result that proved that every non-maximal consistent set can be extended to a maximal-consistent set for a given language (see §4.1), it follows that every

¹²⁴See Chierchia (1985).

partial world can be extended to a possible world. The idea is that, having marked out a formula (or set of formulas) P as relevant, we should be able to restrict our focus to certain other *wffs*. This means that, firstly, we only want to include the relevant P and its relevant implications in a partial world; and, secondly, we want to derive a model consisting of partial worlds that demonstrate all consistent alternative assignments of truth-value to P and its relevant implications in the form of a set of partial worlds. The set of relevant formulas P would normally be the union of the assumption-set A operative in that context and the modal claim ϕ under consideration. The technical details of this process shall not be covered due to space constraints.¹²⁵ However, the basic idea is that a set S is defined, which gives the closure of P under its subformulas, in addition to the closure of P 's subformulas under single negation. S therefore gives the relevant *implications* of a given formula P , restricting the focus to a finite set of implications that pertain only to the atomics of that relevant formula (achieved by identifying the subformulas of P); and S also gives the *alternatives* to P and its relevant implications, by including the different truth-values that can be assigned to the relevant formulas and implications (achieved by identifying the negations).

We may then construct equivalence relations on W as follows: for any $\Gamma, \Delta \in W$, $\Gamma \sim_S \Delta$ *iff* $\forall p \in S : M, \Gamma \Vdash_{v*I} p \leftrightarrow \Delta \Vdash_{v*I} p$. That is, an equivalence relation holds between two worlds if and only if the elements of S true at each of those worlds are identical under the same valuation of variables and interpretation of constants. We then define the equivalence class of Γ , such that $[\Gamma] = \{\Delta \in W : \Gamma \sim_S \Delta\}$. Each equivalence class may be construed as a partial world, since it only assigns truth-values to the set of relevant *wffs* P , and relevant implications of P .

We may then define *partial models* PM_S (models consisting of partial worlds, derived from imposing equivalence classes on a non-partial model M in the described way relative to a set of *wffs* S). We call PM_S a *filtration* of a model M relative to S *iff*, for all formulas $p \in S$ and all $\Gamma \in W$, $PM_S, [\Gamma] \Vdash_{v*I} p \leftrightarrow M, \Gamma \Vdash_{v*I} p$. All of this ensures that there is a partial model PM_S containing a single partial world pw for each equivalence class derived from M , where only the *wffs* of S are assigned truth-values. The truth-conditions for modal claims may then be adapted for these partial models, which allows it to be postulated that individuals only consider a

¹²⁵For more technical details, see Blackburn et al. (2006) pp. 267-8; and Gabbay (1972). Bear in mind that filtration is normally used to prove decidability for modal logics, and I have adapted it for current purposes.

finite number of non-maximal, consistent partial worlds in assigning a truth-value to a modal claim.

This appears to successfully deal with the current objection. It is also worth noting that partial worlds may be used to address the limitation associated with the narrow construal of the relevant assumptions, discussed in the response to Question 3. That is, although some of the non-partial worlds accessible from a context in which the question under discussion concerns Adam and snakes will feature married bachelors, the accessible partial worlds will not feature such bizarre entities, due to the fact that claims about bachelors will not receive truth-values at such partial worlds if those claims are not relevant in the context.

Conclusion

In this thesis, I have developed a reductive account of metaphysical modal claims. It has been argued that a metaphysical modal claim is true *iff* there is a maximal-consistent set of sentences compatible with the contextually-determined assumptions, with essentialist claims playing a significant role in these assumption-sets. In §2, a reductive definition of ‘possible world’ was said to be one that was both non-circular and materially-adequate. The motivation for a reductive account was then found to derive from a limiting primitives principle. An initial problem was establishing a means to tally up the primitives that one’s definition commits one to in a comparative sense. As a solution, it was said that one increases the likelihood of minimising the primitives one is (comparatively) committed by giving a reductive definition that employs only primitives that are difficult to reductively define themselves. §3 involved the development of an argument that only linguistic ersatz accounts have the potential to be reductive. The section concluded with the claim that a linguistic ersatz who endorses the primitives ‘set’ and ‘sentence’ would maximize the likelihood of limiting the primitives he is committed to when comparison with a rival account occurs. This is because the linguistic ersatz’s two primitives have a good claim to being difficult to reductively define, meaning that rival definers are likely to accept them as primitives anyway. §4 consisted of the provision of such a reductive, linguistic ersatz account. This account was argued to be non-circular due to its defining ‘possible world’ in terms of maximal-consistent sets of sentences. It was also argued to be materially-adequate due to the introduction of assumption-relative accessibility relations that appropriately constrain the accessible worlds. It was concluded that the arguments from §2 and §3 provided adequate motivation to adopt this reductive account. Then, in §5, attempts were made to extend the account to accommodate metaphysical modal claims. It was discovered that this could only be achieved by postulating an essentialist predicate that appeared in the assumption-sets of such claims. In §6, it was argued that this essentialist predicate is not a covert modal term, and that it should instead be endorsed as a primitive. However, this has an interesting consequence: endorsing the third primitive ‘essential’ no longer renders it likely that the advocate of this account will (comparatively) commit herself to fewer primitives than a rival. In fact, it turns out that the advocate of this account (comparatively) commits herself to the exact same number of primitives as the modal primitivist who gives a reductive modal definition of ‘essential’! The main motivation for this account *qua* reductive account is therefore undercut. Nevertheless, the achievement

of certain goals set out in the introduction should not go unremarked upon.

The first aim was to provide an account of the meanings and truth-values of metaphysical modal claims within an account of non-metaphysical modal claims. This target has been met, since the exact same machinery of maximal-consistent sets of sentences and assumption-relative modal operators is utilised for all flavours of modality. Secondly, I aimed to develop an approach that reflects the process by which speakers make and assess metaphysical modal claims. I believe that the notion of assumption-relative modal operators reflects the techniques employed in formal semantics to handle non-metaphysical modal claims. I also spent much of §7 suggesting ways to adapt the account to better represent everyday modal discourse, including: extending the account to natural language modal claims, allowing metaphysicians' experiences of modal discourse to influence aspects of the theory, accommodating contextualist and relativist perspectives of the selection of assumption-sets, speculating about the constraints that are imposed on the sorts of things that may be assumed and clarifying a means to convert possible worlds to more tractable partial worlds. The third aim was to produce an account that was as flexible as possible, to allow it to be adapted to reflect diverse perspectives. The topics on which my account remains neutral include: the absolute boundaries of the space of possible worlds (though my preference is to draw them broadly, so as to consider logical possibilities to be legitimate possibilities); the presence of the essentialist predicate \mathbb{E} in the modal predicate language (though my preference is for absence); the existence and content of a privileged set of assumptions for metaphysical modal claims (my preference is to deny the existence of such an assumption-set); the context at which an assumption-set is selected to constrain the interpretation of a modal claim (my preference is for flexible relativism); and the breadth of assumptions operative in the consideration of a particular modal claim (my preference is to keep the assumptions narrow and allow some bizarre worlds to be accessible, though eliminable if one considers only partial worlds). There are a few points of inflexibility, such as the endorsement of a constant domain semantics.

Despite these merits of the account, and the fact that the recommendations of the limiting primitives principle have been successfully followed, the limiting primitives principle does not provide objective motivation to adopt it over a primitivist account of modality. Nevertheless, a subjective motivation, for those who do not find modal terms to be plausible primitives, remains. For such individuals, it is hoped that this account of the meanings and truth-values of metaphysical modal claims will be of some use.

Bibliography

- Adams, R. M. (1974). Theories of actuality. *Nous*, 8.
- Balbiani, P. (1996). Modal logics with relative accessibility relations. volume 1085 of *Lecture Notes in Computer Science*. Springer.
- Ballarin, R. (2005). Validity and necessity. *Journal of Philosophical Logic*, 34.
- Blackburn, P., van Benthem, J., and Wolter, F. (2006). *Handbook of Modal Logic*. Elsevier Science.
- Carnap, R. (1988). *Meaning and Necessity: A Study in Semantics and Modal Logic*. University of Chicago Press.
- Castaneda, H.-N. (1972). Thinking and the structure of the world. *Critica: Revista Hispanoamericana de Filosofia*, 6.
- Chierchia, G. (1985). Formal semantics and the grammar of predication. *Linguistic Inquiry*, 16.
- Chisholm, R. M. (1967). Identity through possible worlds: Some questions. *Nous*, 1.
- Cocchiarella, N. B. (1975). On the primary and secondary semantics of logical necessity. *Journal of Philosophical Logic*, 4.
- Cresswell, M. J. (1972). The world is everything that is the case. *Australasian Journal of Philosophy*, 50.
- Cresswell, M. J. and Hughes, G. E. (2012). *A New Introduction to Modal Logic*. Routledge.
- Divers, J. (2002). *Possible Worlds*. Psychology Press.

- Enderton, H. B. (2001). *A Mathematical Introduction to Logic*. Elsevier Science.
- Fine, K. (1994a). Essence and modality: The second philosophical perspectives lecture. *Philosophical Perspectives*, 8.
- Fine, K. (1994b). Senses of essence. In Sinnott-Armstrong, W., editor, *Modality, Morality and Belief: Essays in Honour of Ruth Barcan Marcus*. Cambridge University Press.
- Fine, K. (2002). Varieties of necessity. In Gendler, T. S. and Hawthorne, J., editors, *Conceivability and Possibility*. Oxford University Press.
- Fitting, M. and Mendelsohn, R. L. (1998). *First-Order Modal Logic*. Springer.
- Gabbay, D. M. (1972). A general filtration method for modal logics. *Journal of Philosophical Logic*, 1.
- Goldblatt, R. (2006). Mathematical modal logic: a view of its evolution. In Gabbay, D. M. and Woods, J., editors, *Handbook of the History of Logic Vol 7*. Elsevier.
- Hintikka, J. (1969). The modes of modality. In *Models for Modalities*, pages 71–86. Springer.
- Kleene, S. C. (2002). *Mathematical Logic*. Dover.
- Knobe, J. and Yalcin, S. (2013). Epistemic modals and context: experimental data. *University of California, Berkeley*.
- Kratzer, A. (1991). *Modality*. de Gruyter.
- Kripke, S. (1963). Semantical analysis of modal logic i: Normal modal propositional calculi. *Mathematical Logic Quarterly*, 9.
- Kripke, S. (1980). *Naming and Necessity*. Harvard University Press.
- Lassiter, D. (2011). *Measurement and modality: The scalar basis of modal semantics*. PhD thesis, New York University.
- Leibniz, G. (1985). In Huggard, E. M., editor, *Theodicy: Essays on the Goodness of God, the Freedom of Man and the Origin of Evil*. Open Court.
- Leung, K. and Chen, D. (1992). *Elementary Set Theory, Part I*. Hong Kong University Press.

- Lewis, D. K. (1986). *On the Plurality of Worlds*. Blackwell.
- Loux, M. J. (1979). Introduction: Modality and metaphysics. In Loux, M. J., editor, *The Possible and the Actual: Readings in the Metaphysics of Modality*. Cornell University Press.
- Lycan, W. G. (1994). *Modality and Meaning*. Springer.
- MacFarlane, J. (2007). Relativism and disagreement. *Philosophical Studies*, 132.
- MacFarlane, J. (2011). Epistemic modals are assessment-sensitive. In Weatherson, B. and Egan, A., editors, *Epistemic Modality*. Oxford University Press.
- Mackie, P. (2006). *How Things Might Have Been*. Oxford University Press.
- Marcus, R. B. (1967). Essentialism in modal logic. *Nous*, 1.
- Parsons, T. (1967). Grades of essentialism in quantified modal logic. *Nous*, 1.
- Portner, P. (2009). *Modality*. Oxford University Press.
- Quine, W. (1951). Ontology and ideology. *Philosophical Studies*, 2.
- Quine, W. V. (1966). Three grades of modal involvement. In *The Ways of Paradox*. Random House.
- Roberts, C. (1996). Information structure in discourse: Towards an integrated formal theory of pragmatics. *Working Papers in Linguistics-Ohio State University Department of Linguistics*, 91.
- Roca-Royes, S. (2011). Essential properties and individual essences. *Philosophy Compass*, 6.
- Rosenkrantz, G. (1993). *Haecceity: An Ontological Essay*. Springer.
- Roy, T. (1995). In defense of linguistic ersatzism. *Philosophical Studies*, 80.
- Shalkowski, S. A. (1994). The ontological ground of the alethic modality. *Philosophical Review*, 103.
- Sider, T. (2003). Reductive theories of modality. In Loux, M. J. and Zimmerman, D. W., editors, *The Oxford Handbook of Metaphysics*. Oxford University Press.

- Tarski, A. (1995). *Introduction to Logic and to the Methodology of the Deductive Sciences*. Courier Dover Publications.
- Wang, J. (2013). From combinatorialism to primitivism. *Australasian Journal of Philosophy*, 91.
- Williamson, T. (2013). *Modal Logic as Metaphysics*. Oxford University Press.